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Effect of Voltage Sag on Transformers Inrush Current

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Abstract

In this paper, the inrush current due to symmetrical and unsymmetrical voltage sag is studied analytically and in details and considering the fault removal in all three phases simultaneously. This summarization leads to the determination of analytical phrases of magnetic flux and inrush current, after the voltage sag. The dependence of the inrush current to type, depth, duration and the initial point of the sag are determined. These phrases describe the behavior of three phase transformers during the voltage recovery after the voltage sag and an approximate value for inrush current peak will be obtained analytically. These values can be useful in power system studies such as relays synchronization, transformers tension and etc.

Keywords: Transformer, Inrush Current, Voltage sag.

1. Introduction

The transformers are one of the basic and expensive equipments of electrical networks, which their repairing is very costly and time consuming. On the other hand, occurring any kind of error in performance of these equipments, leads to long term energy outage, and consequently reducing the income of electric generation and distribution companies and dramatic reduction in reliability of electric system. The phenomenon of inrush current in transformers is one of the events that cause the transformer error and consequently disconnection of continuous flow of the electric power transmission. The voltage sag is a temporary reduction in the rms value of voltage in one or several phases. Voltage sags have

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been considered because they cause problems in the equipments such as speed regulation drives, computers, and industrial control systems [1]. For the power transformers, when a fault is removed, a sudden voltage recovery can saturate the transformer core. This core saturation can include high inrush currents [2] that are sensitive to the moment of voltage recovery. This moment can only have discrete values, because the fault removal is done, only in natural current zeros. However, the assuming of the voltage removal in all phases simultaneously, simplify the study and provide an initial estimation of the inrush current. By assuming this fact that the voltage recovery can only be done in current zeros, the calculated current peaks are less than the obtained current peaks, when this assumption has not been considered. In [3], a nonlinear model of a three-phase three-leg transformer, is studied by PSpice software to study the effective variables on the peak value of inrush current such as depth, duration of voltage sag and the initial point on waveform, due to symmetrical voltage sag. In [4], the effect of symmetrical and unsymmetrical voltage sags in the three-phase transformers is widely studied.

In a research in 2006 and by numbers of scientist in turkey, the effect of phase jump on the inrush current in a transformer, due to voltage sag recovery is studied. The short circuit or fault in a system, in addition to cause voltage sag in the system, leads to the change of phase angle (it means that the angle before the voltage sag and the angle during voltage sag are different) [5]. In another research, researchers studied the effects of different kinds of symmetrical and unsymmetrical voltage sags on three-phase transformers by using computer simulation of MATLAB/SIMULINK software and also studied the effects of different parameters of voltage sag such as initial point on wave, duration and depth of voltage sag [6]. In [7], the performance of an unsymmetrical and nonlinear three-phase three-leg transformer under the voltage swell is studied and the voltage and current waveforms for different types of voltage swells with different magnitudes and duration is calculated. It should be noted that the voltage swells are the disturbances with very short durations that

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leads to increase in rms voltage. They create by reasons such as too mush excitation in system due to too mush reactive power injection, capacitance bank switching and exciting, disconnecting loads, resonance in system and different kinds of faults. In this paper, the inrush current due to symmetrical and unsymmetrical voltage sag is studied analytically and in details and considering the fault removal in all three phases simultaneously is studied. This summarization leads to the determination of analytical phrases of magnetic flux and inrush current, after the voltage sag. The dependence of the inrush current to type, depth, duration and the initial point of the sag are determined. These phrases explain the behavior of three-phase transformers during the voltage recovery after the voltage sag and an approximate value for inrush current peak will be obtained analytically. These values can be useful in power system studies such as relays synchronization, transformers tension and etc.

2. Classification of Voltage Sags

IEEE and IEC do not present any standard definition of magnitude indexes for three-phase events. So, in published papers, the sag magnitude for three-phase events has been shown in different forms such as the worst phase [8], characteristic voltage [1], or the symmetrical components of three-phase voltage [9]. Every form of three-phase event definition by magnitude has its own advantages in its own domain. As it was noted, there is no general agreement about choosing one of them as an index for three-phase voltage sag, until now. In this paper, the ABC classification form [1] has been used, because understanding different types of voltage sags is much easier through it. The ABC form emphasizes on the experienced voltage sags by final user equipments and includes seven types of three-phase voltage sags. In this paper, four types of voltage sags that are proportional to four types of faults in power system, has been considered, that are as follows:

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- **Voltage sag type A:** Symmetrical three-phase fault.
- **Voltage sag type B:** Single phase to ground fault.
- **Voltage sag type C:** Two-phase fault.
- **Voltage sag type E:** Two-phase to ground fault.

3. Transformer Model

A three-phase three-leg transformer has been considered in this paper and the proposed model in [3] and [4] is used to study this transformer.

3.1. The Electric and Magnetic Circuit Models

The electric and magnetic equations of Yg-Yg three-phase transformers have been shown in Figure . (1) and (2):

$$\begin{aligned} u_{pk} &= \left(R_p + L_{dp}\frac{d}{dt}\right)i_{pk} + N_p\frac{d\varphi_k}{dt} \\ u_{sk} &= \left(R_s + L_{ds}\frac{d}{dt}\right)i_{sk} + N_s\frac{d\varphi_k}{dt}, \quad (k = a,b,c) \\ N_p i_{pek} &+ N_s i_{sk} = i_{mk} = f_k - f_d \\ \varphi_a &+ \varphi_b + \varphi_c + \varphi_d = 0 \end{aligned} \tag{1}$$

In this model every leg is seen as a separate magnetic element, and the phrase that is used for nonlinear behavior of the core is a $\mathrm{R}_k(f_k)$ function that relates the magnetic potential in leg and the flux through it ($f_k = \mathrm{R}_k(f_k).\varphi_k$).

$$\mathrm{R}_k(f_k)^{-1} = K_{1k}\left(1 + \left(\frac{|f_k|}{f_{0k}}\right)^{p_k}\right)^{-1/p_k} + K_{2k} \tag{2}$$

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Where K_{1k} ، K_{2k} ، p_k and f_{0k} are experimental parameters that allow to the single-value function to be fitted to the saturation curve of transformer ($\varphi_k - f_k$) [3].

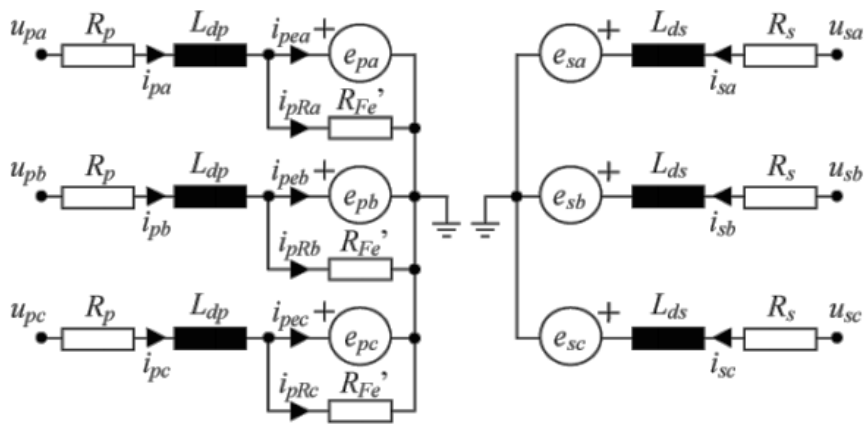

Figure ure (1): Electric equivalent circuit of a three-phase Yg-Yg transformer

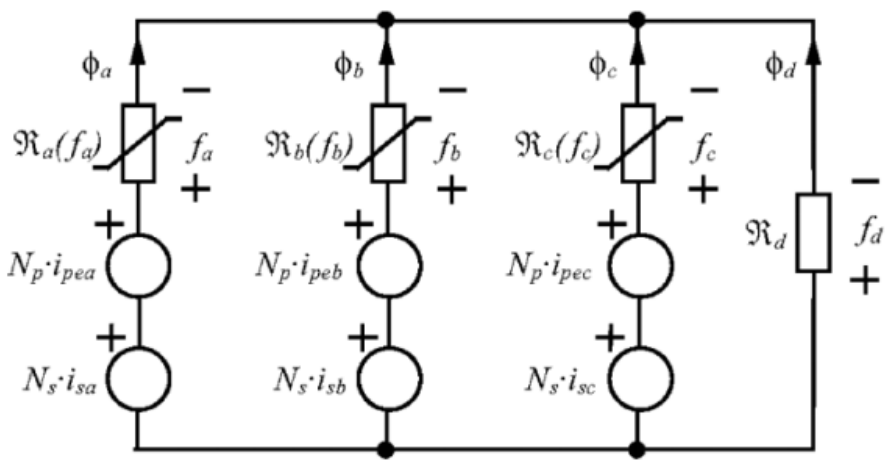

Figure (2): Magnetic equivalent circuit of a three-phase three-leg transformer

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It can be seen in Figure (2) That the reluctance of R_d is assumed constant, because it shows an air path in a three-leg transformer.

For separating the number of winding turns, the above relations are rewritten according to [4]:

$$\begin{aligned} u_{pk} &= \left(R_p + L_{dp}\frac{d}{dt}\right)i_{pk} + \frac{d\lambda_{pk}}{dt} \\ u_{sk} &= \left(R_s + L_{ds}\frac{d}{dt}\right)i_{sk} + \frac{1}{r_{t,w}}\frac{d\lambda_{pk}}{dt}, \quad (k = a,b,c) \\ i_{pek} &+ i_{sk} / r_{t,w} = i_{mk} = f'_k - f'_d \\ \lambda_{pa} &+ \lambda_{pb} + \lambda_{pc} + \lambda_{pd} = 0 \end{aligned} \tag{4}$$

Where $r_{t,w} = N_p / N_s$ is winding turn ratio and $\lambda_{pk} = N_p \varphi_k$ is the core magnetic flux linked by the primary windings. The relation between the magnetic potential in the leg and the flux through it is rewritten as $f'_k = R'_k(f'_k).\lambda'_k$.

Because in here, only the primary voltages, currents and fluxes are considered, these primary variables are rewritten as follows for more clarity:

$$u_k = u_{pk}, \quad i_k = i_{pk}, \quad \lambda_k = L_{dp} i_{pk} + \lambda_{pk} \tag{4}$$

2.3. Transformer Data

The study of the three-leg model is performed with a three-phase Yg-Yg, 60 kVA, 380/220 V transformer witch its linear and nonlinear parameters have been obtained by laboratory measurements in [4]. These parameters are given in Table (1). In every studied cases in this paper, the secondary side supply a resistive 0.8 pu rated load. In fact, the magnitude and the type of load (resistive or inductive) have no effect on the transformer behavior [3].

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Table (1): The data of three-leg three-phase transformers

60 kVA, 380/220 V, 50 Hz, Wye G-Wye G transformer				
Winding resistances and leakage inductance:				
$R_p = 0.0036\ \Omega\ \ (0.0015\ pu)$		$R_s = 0.0012\ \Omega\ \ (0.0015\ pu)$		
$L_{dp} = 0.1524\ mH\ \ (x_{dp} = 0.02\ pu)$		$L_{ds} = 0.0511\ mH\ \ (x_{ds} = 0.02\ pu)$		
Non-linear leg reluctances				
	K'_1 , $Wb/A.t$	K'_2 , $Wb/A.t$	p	f'_0 , $A.t$
$R'_a(f'_a)$	0.3333	0.0001	0.909	**4.05**
$R'_b(f'_b)$	1.4444	0.0002	0.93	**0.9**
$R'_c(f'_c)$	0.3333	0.0001	0.902	**3.973**
Air path linear reluctance and core-loss resistance				
$R'_d = 1300\ A.t/Wb\ \ (3128.7\ pu)$		$R'_{Fe} = 277\ \Omega\ \ (115.10\ pu)$		

4. Obtaining the Total Magnetic Flux

The Ref. [3] and [4] show that the main reason for creation of transformer peak current is the DC component of total magnetic flux after the sag. For example, $\lambda^{(3)}_{k,DC}$ with $k = a,b,c$ in Figure . (3). The transformer saturation occurs when the DC component is not equal to zero, which leads to high current. This section present an analytical study of total magnetic flux, when the voltage sag leads to $\lambda^{(3)}_k$ and $i^{(3)}_k$. By considering the transformer model (3), the total magnetic flux linked with primary winding can be calculated as follows:

$$u_k(t) = R_p i_k(t) + \frac{d\lambda_k(t)}{dt} \approx \frac{d\lambda_k(t)}{dt} \tag{5}$$

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Where primary total magnetic flux is $\lambda_k = L_{dp} i_k + \lambda_{pk}$ and the voltage drop in primary winding resistances has been not considered.

The total magnetic flux is obtained from (5), and is as follows:

$$\int_0^t \lambda_k(t)dt = \int_0^t u_k(t)dt \;\Rightarrow\; \lambda_k(t) = \lambda_k(0) + \int_0^t u_k(t)dt \tag{6}$$

When the supply voltage is $u_k(t) = v_k(t) = \sqrt{2}V_k\, sin(\omega t + \alpha_k)$ with $k = a,b,c$ and the transformer is in the steady state, the magnetic flux is:

$$\lambda_k(t) = \lambda_k(0) + \frac{\sqrt{2}V_k}{\omega}\left[sin\left(\omega t + \alpha_k - \frac{\pi}{2}\right) - sin\left(\alpha_k - \frac{\pi}{2}\right)\right] \tag{7}$$

Since the mean value of flux in steady state is zero, so the next phrase for primary flux will be obtained:

$$\lambda_k(0) - \frac{\sqrt{2}V_k}{\omega} sin\left(\alpha_k - \frac{\pi}{2}\right) = 0 \tag{8}$$

And the final phrase for flux:

$$\lambda_k(t) = \frac{\sqrt{2}V_k}{\omega} sin\left(\omega t + \alpha_k - \frac{\pi}{2}\right) \tag{9}$$

When a sag is created the supply voltages are as follows:

$$u_k(t) = \begin{cases} v_k(t) = \sqrt{2}V_k\, sin(\omega t + \alpha_k), & 0 \le t \le t_i \\ v_{kh}(t) = \sqrt{2}V_{kh}\, sin(\omega t + \alpha_{kh}), & t_i \le t \le t_f \\ v_k(t) = \sqrt{2}V_k\, sin(\omega t + \alpha_k), & t \ge t_f \end{cases} \tag{10}$$

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The pre and post sag voltages with magnitudes $V_a = V_b = V_c = V$ and phase angles $\alpha_a = 0$, $\alpha_b = -2\pi/3$ and $\alpha_c = 2\pi/3$ is defined.

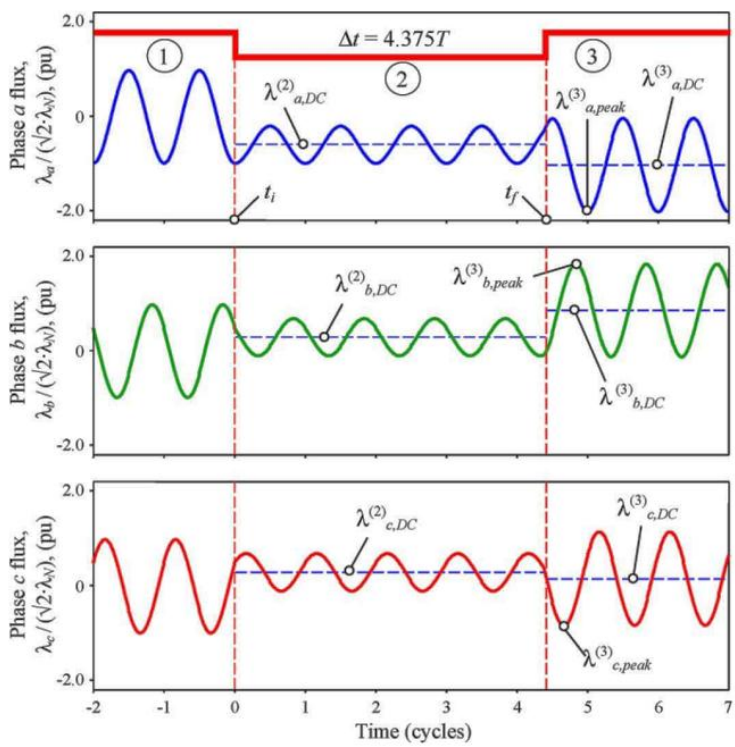

Figure (3): The magnetic fluxes in symmetrical voltage sag (type A) with $h = 0.4$ ، $\Delta t = 4.375T$ **and** $\psi_i = 0°$

The magnitude and angle of voltage phasors during the sag (V_{kh} and α_{kh}) has been shown in table (2). These values have been determined from the sag phrases of table (2) from Ref [4], which has been obtained from sag classification in Ref [1]. In this table *h* and *V* are the sag depth and phase voltage respectively, which $h = 0 \cdots 1$ and $V = U_L / \sqrt{3}$.

The flux continuity shows that the flux can be calculated in each moment from equation (6).

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$$\lambda_k^{(3)}(t)=\lambda_k(0)+\int_0^{t_i} v_k(t)dt+\int_{t_i}^{t_f} v_{kh}(t)dt+\int_{t_f}^{t} v_k(t)dt \tag{11}$$

Where the sag generates between the t_i and t_f moments.

Table (2): The equations of voltage phasors in different kinds of sag [4]

Type A		Type B
$\underline{V}_{ah}=hV$ $\underline{V}_{bh}=-\frac{1}{2}V-j\frac{\sqrt{3}}{2}V$ $\underline{V}_{ch}=-\frac{1}{2}V+j\frac{\sqrt{3}}{2}V$		$\underline{V}_{ah}=hV$ $\underline{V}_{bh}=-\frac{1}{2}hV-j\frac{\sqrt{3}}{2}hV$ $\underline{V}_{ch}=-\frac{1}{2}hV+j\frac{\sqrt{3}}{2}hV$
Type E		**Type C**
$\underline{V}_{ah}=V$ $\underline{V}_{bh}=-\frac{1}{2}hV-j\frac{\sqrt{3}}{2}hV$ $\underline{V}_{ch}=-\frac{1}{2}hV+j\frac{\sqrt{3}}{2}hV$		$\underline{V}_{ah}=V$ $\underline{V}_{bh}=-\frac{1}{2}V-j\frac{\sqrt{3}}{2}hV$ $\underline{V}_{ch}=-\frac{1}{2}V+j\frac{\sqrt{3}}{2}hV$

By considering:

$$\begin{aligned}&\int_0^{t} v_k(t)dt=\int_0^{t_i} v_k(t)dt+\int_{t_i}^{t_f} v_k(t)dt+\int_{t_f}^{t} v_k(t)dt\\&\Rightarrow \int_0^{t_i} v_k(t)dt+\int_{t_f}^{t} v_k(t)dt=\int_0^{t} v_k(t)dt-\int_{t_i}^{t_f} v_k(t)dt\end{aligned} \tag{12}$$

The flux after the sag can be rewritten as follows:

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$$\lambda_k^{(3)}(t)=\lambda_k(0)+\int_0^t v_k(t)dt+\int_{t_i}^{t_f}\left(v_{kh}(t)-v_k(t)\right)dt \tag{13}$$

Which the two first sentences are the same as the sentences for the case without the sag (relation (9)).

So the final phrases for the flux, when the voltage is recovered are:

$$\lambda_k^{(3)}(t)=\frac{\sqrt{2}V_k}{\omega}sin\left(\omega t+\alpha_k-\frac{\pi}{2}\right)+\lambda_{k,DC}^{(3)}(t) \tag{14}$$

Where:

$$\lambda_{k,DC}^{(3)}(t)=\int_{t_i}^{t_f}\left(v_{kh}(t)-v_k(t)\right)dt \tag{15}$$

This sentence is constant and its physical meaning shows a DC magnetic flux. In the following, the $\lambda_{k,DC}$ obtained from (15) is calculated for different kinds of sags.

4.1. The DC component of The Transformer Total Magnetic Flux

The transformer total magnetic flux phrases during the recovery, can be obtained from (15) by considering $v_k(t)=\sqrt{2}V_k\,sin(\omega t+\alpha_k)$ and $v_{kh}(t)=\sqrt{2}V_{kh}\,sin(\omega t+\alpha_{kh})$ with $k=a,b,c$.

$$\begin{aligned}\lambda_{k,DC}=\frac{\sqrt{2}}{\omega}\Bigg\{&V_k\left[sin\left(\omega t_i+\alpha_k-\frac{\pi}{2}\right)-sin\left(\omega t_f+\alpha_k-\frac{\pi}{2}\right)\right]\\&-V_{kh}\left[sin\left(\omega t_i+\alpha_{kh}-\frac{\pi}{2}\right)-sin\left(\omega t_f+\alpha_{kh}-\frac{\pi}{2}\right)\right]\Bigg\}\end{aligned} \tag{16}$$

Which can be rewritten as follows?

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$$\lambda_{k,DC} = \frac{2\sqrt{2}}{\omega} sin\left(\frac{\omega\left(t_i - t_f\right)}{2}\right)\left\{V_k\, sin\left(\frac{\omega\left(t_i + t_f\right)}{2} + \alpha_k\right) - V_{kh}\, sin\left(\frac{\omega\left(t_i + t_f\right)}{2} + \alpha_{kh}\right)\right\} \tag{17}$$

The equation (17) can be rewritten as follows by using duration and initial point on wave (ψ_i):

$$\lambda_{k,DC} = \frac{2\sqrt{2}}{\omega} sin\left(-\frac{\omega\Delta t}{2}\right)\left\{V_k\, sin\left(\psi_i + \frac{\omega\Delta t}{2} + \alpha_k\right) - V_{kh}\, sin\left(\psi_i + \frac{\omega\Delta t}{2} + \alpha_{kh}\right)\right\} \tag{18}$$

So, for different kinds of the sag, the DC component of the total magnetic flux can be obtained from phrase (18) and by considering $V_a = V_b = V_c = V$ and the phase angles $\alpha_a = 0$ ، $\alpha_b = -2\pi/3$ ، $\alpha_c = 2\pi/3$ and the values of V_{kh} and α_{kh} from table (2).

4.1.1. $\lambda_{k,DC}$ In Type A Sag

By replacing the V_{kh} and α_{kh} values for type A sag from table (2) , in relation (18):

$$\lambda_{k,DC}^{(A)} = \frac{2\sqrt{2}V(1-h)}{\omega} sin\left(-\frac{\omega\Delta t}{2}\right) \times sin\left(\psi_i + \frac{\omega\Delta t}{2} + \alpha_k\right), \quad (k = a,b,c) \tag{19}$$

4.1.2. $\lambda_{k,DC}$ In Type B Sag

By replacing the V_{kh} and α_{kh} values for type B sag from table (2) , in relation (18):

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$$\lambda_{a,DC}^{(B)} = \frac{2\sqrt{2}V(1-h)}{\omega} sin\left(-\frac{\omega\Delta t}{2}\right) \times sin\left(\psi_i + \frac{\omega\Delta t}{2}\right)$$
$$\lambda_{b,DC}^{(B)} = \lambda_{c,DC}^{(B)} = 0 \tag{20}$$

4.1.3. $\lambda_{k,DC}$ In Type C Sag

By replacing the V_{kh} and α_{kh} values for type C sag from table (2) , in relation (18):

$$\lambda_{a,DC}^{(C)} = 0$$
$$\lambda_{k,DC}^{(C)} = \frac{2\sqrt{2}V}{\omega} sin\left(-\frac{\omega\Delta t}{2}\right) \times \left\{ sin\left(\psi_i + \frac{\omega\Delta t}{2} + \alpha_k\right) - \frac{\sqrt{1+3h^2}}{2} sin\left(\psi_i + \frac{\omega\Delta t}{2} + \alpha_{kh}\right)\right\}, \quad (k=b,c) \tag{21}$$

By replacing α_{kh} and using the trigonometric relationships, the above phrase can be rewritten as follows:

$$\lambda_{a,DC}^{(C)} = 0$$
$$\lambda_{b,DC}^{(C)} = -\lambda_{c,DC}^{(C)} = \frac{\sqrt{3}}{2}\frac{2\sqrt{2}V(1-h)}{\omega} sin\left(-\frac{\omega\Delta t}{2}\right) \times sin\left(\psi_i + \frac{\omega\Delta t}{2} - \frac{\pi}{2}\right) \tag{22}$$

4.1.4. $\lambda_{k,DC}$ In Type C Sag

By replacing the V_{kh} and α_{kh} values for type E sag from table (2) , in relation (18):

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$$\lambda_{a,DC}^{(E)} = 0$$
$$\lambda_{k,DC}^{(E)} = \frac{2\sqrt{2}V(1-h)}{\omega} sin\left(-\frac{\omega\Delta t}{2}\right) \times sin\left(\psi_i + \frac{\omega\Delta t}{2} + \alpha_k\right), \quad (k=b,c) \tag{23}$$

5. Obtaining The Analytical Relations of Current Peaks

If the magnetic characteristic of all the transformer legs is the same, then the current peak generates while a flux peak exists, because they relate to $\lambda - i$ magnetic curve. But when the transformer legs have different magnetic characteristics, the maximum inrush current of each phase should be calculated by using maximum magnetic flux of each path and the nonlinear relation of magnetic path, and then the maximum value of maximum phases should be considered as inrush current peak. So, the maximum values of total magnetic fluxes should be calculated in order to determining the current peaks.

The maximum of sag generated flux in each phase depends on the depth h, duration Δt, and the initial point on wave ψ_i. So the following function should be calculated:

$$\lambda_{k,max}(h,\Delta t,\psi_i) = max\left\{\left|\lambda_k^{(3)}(t)\right|\right\} = \frac{\sqrt{2}V}{\omega} + \left|\lambda_{k,DC}(h,\Delta t,\psi_i)\right|, \quad (k=a,b,c) \tag{24}$$

Where the $\lambda_{k,DC}(h,\Delta t,\psi_i)$ values for different types of sag, has been calculated as analytical phrases in previous section.

5.1. Recognition of The Current Peak

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If simplification in the transformer model is performed as follows, the total flux and primary currents of transformer can be related to each other.

- The transformer is in the no load mode (or in the saturation mode, the secondary currents have less importance than the primary currents) ($i_{sk} \approx 0$ or $i_{sk} \square\ i_{pk}$)
- The core loss equivalent resistances are ignored ($i_k = i_{pek}$)
- In this mode the magnetic equation (3) leads to:

$$i_{pek} + i_{sk} / r_{t,w} = i_{mk} = f'_k - f'_d \rightarrow i_k = f'_k = \lambda_{pk} \mathrm{R}'_k(i_k) \tag{25}$$

From this phrase, the $\lambda_k - i_k$ relation is calculated:

$$\lambda_k = L_{dp} i_{pk} + \lambda_{pk} = \left(L_{dp} + \mathrm{R}'_k(i_k)^{-1}\right) i_k \tag{26}$$

And replacing the nonlinear phrase $\mathrm{R}'_k(f'_k)$ in (3) leads to:

$$\lambda_k = \left(L_{dp} + K'_{1k}\left(1 + \left(\frac{i_k}{f'_{0k}}\right)^{p_k}\right)^{-1/p_k} + K'_{2k}\right) i_k \tag{27}$$

The above nonlinear relation can be approximated by piecewise - linear saturation curve. If only two pieces is used, their parameters can easily be related to nonlinear function parameters (27). The two pieces of the two piece linear saturation curve is defined as follows:

$$\begin{aligned} &if\ \lambda_k < \lambda_{0k} \Rightarrow \lambda_k = \left(L_{dp} + K'_{1k} + K'_{2k}\right) i_k \\ &if\ \lambda_k > \lambda_{0k} \Rightarrow \lambda_k = K'_{1k} f'_{0k} + \left(L_{dp} + K'_{2k}\right) i_k \end{aligned} \tag{28}$$

Where $\lambda_{0k} = \left(L_{dp} + K'_{1k} + K'_{2k}\right) f'_{0k}$ is the saturation flux. The value of inrush current can be easily calculated by (28).

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The determination of inrush current from the saturation curve is the same as the graphic method of calculating inrush current during transformer energizing. The only difference is that in here, the sag arbitrary parameter (h ، Δt or ψ_i) is used instead of time.

By considering (28), the inrush current phrase is obtained as follows:

$$\begin{aligned} &if\ \lambda_k < \lambda_{0k} \ \Rightarrow\ i_k = \frac{\lambda_k}{L_{dp} + K'_{1k} + K'_{2k}} \\ &if\ \lambda_k > \lambda_{0k} \ \Rightarrow\ i_k = \frac{\lambda_k - K'_{1k} f'_{0k}}{L_{dp} + K'_{2k}} \end{aligned} \tag{29}$$

By considering sag type and calculating the maximum flux of each phase, and then obtaining the inrush current in each phase, the transformer inrush current peak can be calculated in any situation.

6. The Study of Behavior of Three-phase Three-leg Transformer

The previous section phrases, enable the calculation of the transformer current peak. So, the effect of type, depth h, duration Δt, and initial point on wave ψ_i, the sag on the current peaks can be calculated. It should be mentioned that, in the previous section study, the source does not have any internal impedance. This fact results in overestimating the transformer current peak, because the source impedance damps these peaks [3].

6.1. The Variation Procedure of the Inrush Current Peak with Δt Duration and ψ_i Initial Point on Wave

For simultaneous study of the effect of sag duration Δt and initial point on wave ψ_i, on the inrush current peaks, the peak value of inrush current for the A,B,C and E sag types with

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$h = 0.4$ ، $4T \le \Delta t \le 6T$ and $0° \le \psi_i \le 360°$ has been obtained and has been shown in Figure ures (4) to (7) respectively.

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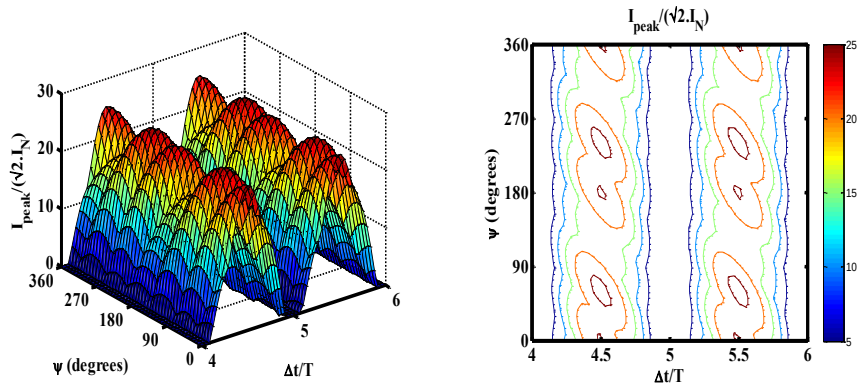
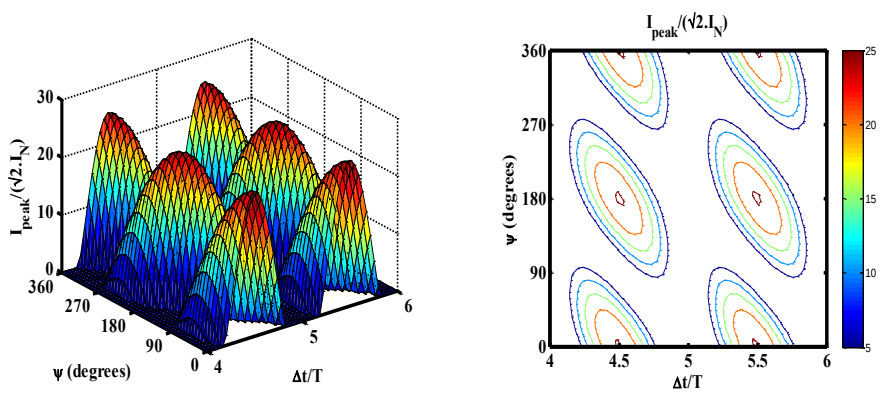


(a) (b)

Figure . (4): The inrush current peak for sag type A per Δt and ψ_i ($h = 0.4$)



(a) (b)

Figure . (5): The inrush current peak for sag type B per Δt and ψ_i ($h = 0.4$)

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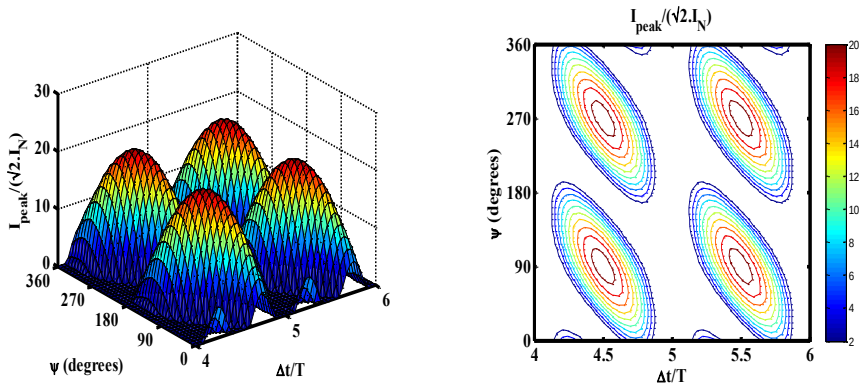
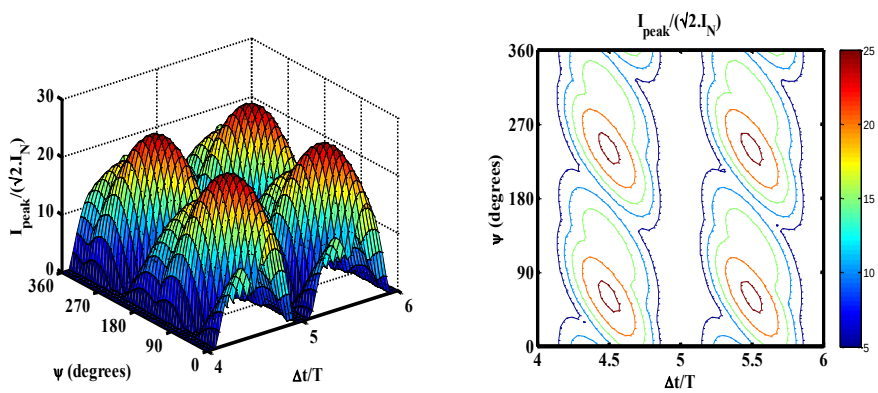


(a) (b)

Figure . (6): The inrush current peak for sag type C per Δt and ψ_i ($h = 0.4$)



(a) (b)

Figure . (7): The inrush current peak for sag type B per Δt and ψ_i ($h = 0.4$)

6.2. The Variation Procedure of the Inrush Current Peak with Δt Duration and ψ_i Initial Point on Wave

For simultaneous study of the effect of sag depth h and sag duration Δt, on the inrush current peaks, the peak value of inrush current for the A,B,C and E sag types with $0 \le h \le 1$ ،

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$4T \le \Delta t \le 6T$ and $\psi_i = 0°$ has been obtained and has been shown in Figure ures (8) to (11) respectively.

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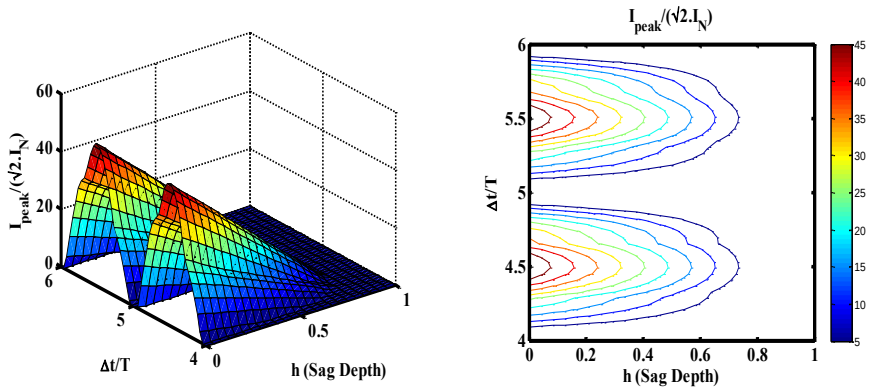
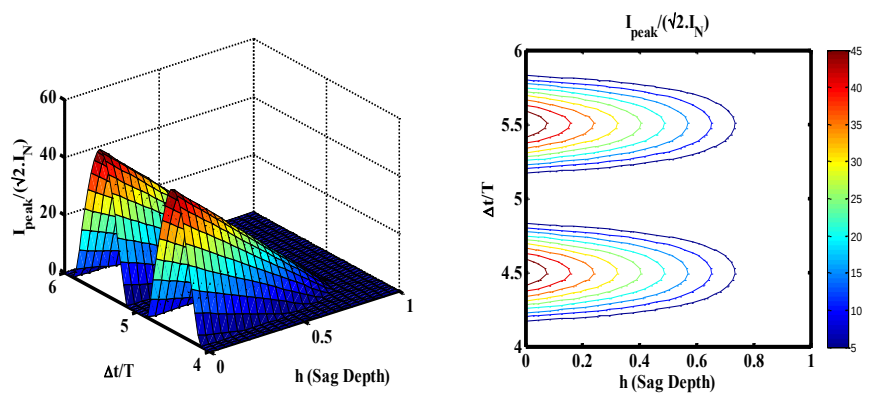


(a) (b)

Figure . (8): The inrush current peak for sag type A per h and Δt ($\psi_i = 0°$)



(a) (b)

Figure . (9): The inrush current peak for sag type B per h and Δt ($\psi_i = 0°$)

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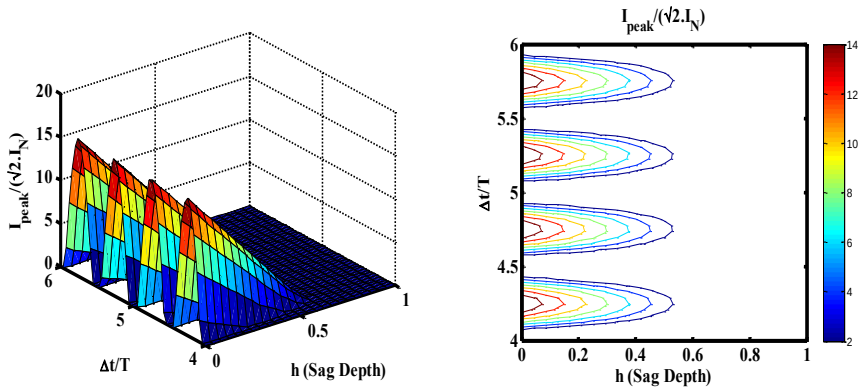
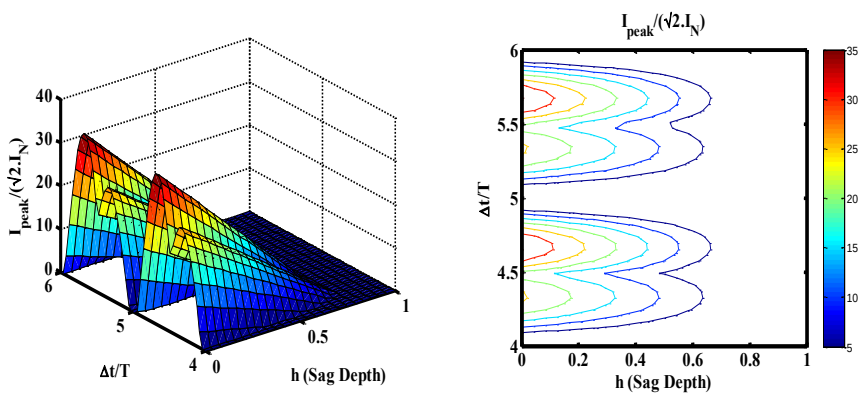


(a) (b)

Figure . (10): The inrush current peak for sag type C per h and Δt ($\psi_i = 0°$)



(a) (b)

Figure . (11): The inrush current peak for sag type E per h and Δt ($\psi_i = 0°$)

6.3. The Variation Procedure of the Inrush Current Peak with Δt Duration and ψ_i Initial Point on Wave

For simultaneous study of the effect of sag depth h and initial point on wave ψ_i, on the inrush current peaks, the peak value of inrush current for the A,B,C and E sag types with $0 \le h \le 1$،

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$0° \le \psi_i \le 360°$ and $\Delta t = 5.5T$ has been obtained and has been shown in Figure ures (12) to (15) respectively.

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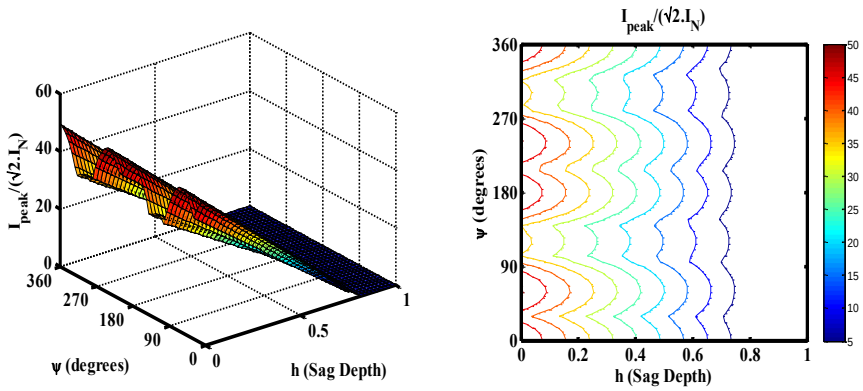
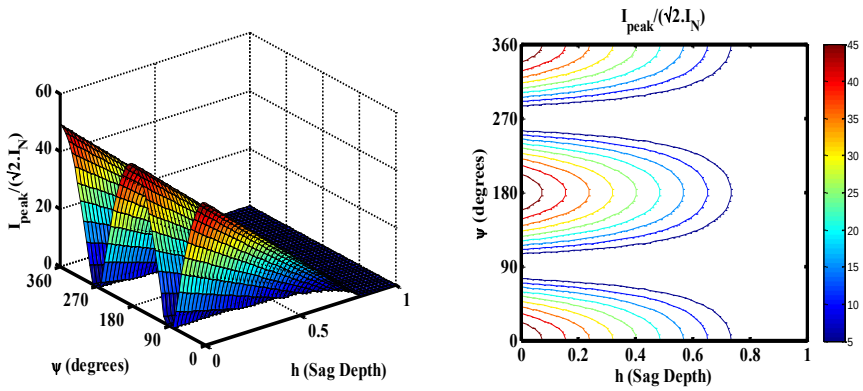


(a) (b)

Figure . (12): The inrush current peak for sag type A per h and ψ_i ($\Delta t = 5.5T$)



(a) (b)

Figure . (13): The inrush current peak for sag type B per h and ψ_i ($\Delta t = 5.5T$)

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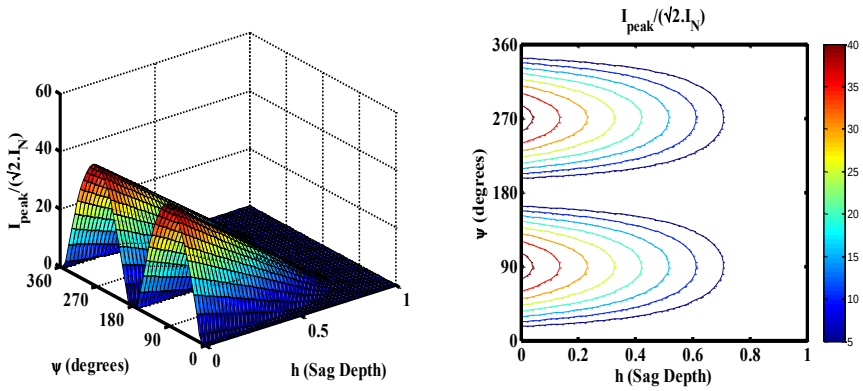
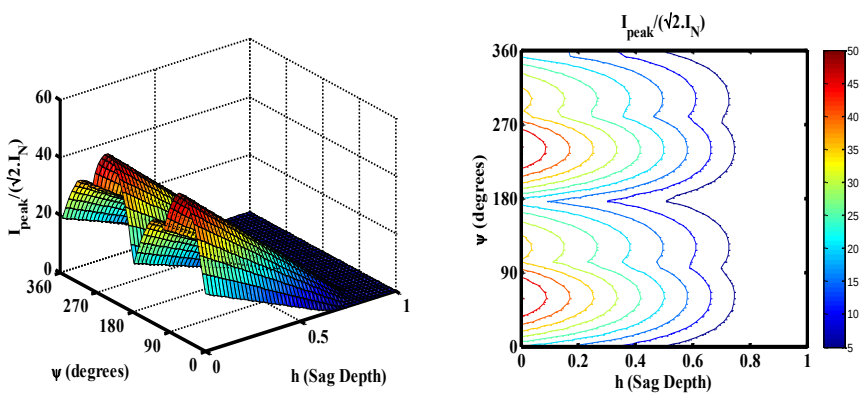


(a) (b)

Figure . (14): The inrush current peak for sag type C per h and ψ_i ($\Delta t = 5.5T$)



(a) (b)

Figure . (15): The inrush current peak for sag type E per h and ψ_i ($\Delta t = 5.5T$)

6.4. The Simulation Results Discussion

By comparing the maximum values of the inrush current peak and ψ_i and Δt values in Figure . (4) With the corresponding values of this Figure ure in the corresponding Figure ures

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in [3], it can be concluded that the analytical study, predict the exact behavior of three-leg transformer. Despite this, the analytical results are a bit less that the simulation results in [3], because the resistance voltage drop has been ignored.

Considering Figure ures (11) to (24), it is obvious that in all types of sags, the dependence of inrush current peaks after the voltage sag, to the sag depth is linear. The other conclusion that can be extracted by study of these voltage sags, is that if the sag depth is more than a particular value, the inrush current peak per all of the ψ_i and Δt values, has very small magnitude and by becoming smaller than this particular value, like the previous results, the inrush current peak magnitude increase linearly.

Considering the extracted diagrams, the inrush current peaks are repeated per every 180 degree changing in ψ_i. In the other words, if the h and Δt are kept constant, the inrush current peak variations per ψ_i, show a periodic pattern with the period of 180 degree. Considering $\psi_i = \omega t_i = \frac{360}{T} t_i$ (per degree), it can be concluded that the period of the variation of the inrush current peak per equals to half of the basic waveform period. It is because of the fact that by passing a half cycle, the voltage values become symmetry; but considering the equality of the voltage absolute values, the DC value of the generated flux in rotor, is the same as its previous cycle (in opposite direction), so the generated current peak in these two states are the same (the currents are in opposite directions in these two states). These conclusions also can be extracted from the DC flux relations that are obtained previously.

Also by keeping h and ψ_i constant, the variation of inrush current peaks per Δt, show a periodic pattern with a period equal to the period of the basic three-phase waveform (T). This conclusion is also can be expected from DC flux relations.

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The other conclusion that can be extracted from simulations is that per values of Δt which is the integer multiple of the T, the current peaks will be minimum (zero). The reason is that the generated DC flux during the voltage recovery moment is zero.

In all types of voltage sags, for the different values of the parameters Δt and ψ_i, the values of the other parameter, that maximize the current peak, is changed, and is not unique. However, for each type of sag and per every arbitrary value of h, there are particular values of Δt and ψ_i pairs that maximize the inrush current peaks $(\Delta t_{max}, \psi_{i-max})$. These ordered pairs, that show the worst case possible for inrush current due to voltage recovery, along with the maximum value of current peaks per $h = 0.4$ have been shown in table (3).

Table (3): The Δt and ψ_i values that maximize the inrush current peak along with the inrush current peaks maximum values per $h = 0.4$

$I_{Peak-max} / \left(\sqrt{2} I_n\right)$ per $h = 0.4$	$(\Delta t_{max}, \psi_{i-max})$	Sag Type
26.08	$(2nT + T, 180k + 60°), \quad n,k \in \square$	A
25.29	$(2nT + T, 180k°), \quad n,k \in \square$	B
21.20	$(2nT + T, 180k + 90°), \quad n,k \in \square$	C
26.08	$(2nT + T, 180k + 60°), \quad n,k \in \square$	E

Considering table (3), the worst case possible is directly depends on the Δt and ψ_i parameters. So, by finding the values of these parameters that maximize the DC flux after the

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voltage recovery (from the obtained relations), the approximate values for $(\Delta t_{max}\ ,\ \psi_{i-max})$ can be determined. However, it should be mentioned that despite the obvious dependence of the DC flux to Δt and ψ_i, the inrush current peak maximum does not necessarily corresponds to the DC flux maximum. This phenomenon is due to the difference of the magnetic curve of the different legs. It means that in a particular condition, because of bigger reluctance in a leg, despite the higher value of the flux in a leg, the inrush current peak of that leg is smaller.

It is also interesting that in the worst case, the magnitude of the inrush current peak after the voltage recovery of a sag with 0.4 depth, for all type of sags, is bigger than 20 times of the rated current of the transformer. So consideration of this phenomenon is necessary in protection system of transformer, and methods should be developed for distinction of this current from the fault current.

7. Conclusion

In this paper, the inrush current due to symmetrical and unsymmetrical voltage sag is studied analytically and in details and considering the fault removal in all three phases simultaneously, and the analytical phrases for magnetic flux and inrush current after the voltage sag is obtained. The dependence of the inrush current to the type, depth, duration and initial point on wave is determined. Thus, the behavior of three-phase transformers during the voltage recovery after a sag is explained and an approximate value for inrush current peak is obtained analytically. These values can be useful for the study of power systems such as relays synchronization, transformers tension and etc.

References:

[1] Bollen, Math HJ. “Understanding power quality problems”. Vol. 3. New York: IEEE press, 2000.

Vol. 4(12), Jul, 2014, pp. 811-839, **ISSN**: 2305-0543
Available online at: http://www.aeuso.org

[2] E. Styvaktakis, M. H. J. Bollen, and I. Y. H. Gu, "Transformer saturation after a voltage dip," *IEEE Power Eng. Rev.*, vol. 20, no. 4, pp. 62–64, Apr. 2000.

[3] L. Guasch, F. Córcoles, J. Pedra, and L. Sáinz, "Effects of symmetrical voltage sags on three-phase three-legged transformers," I*EEE Trans. Power Del.*, vol. 19, no. 2, pp. 875–883, Apr. 2004.

[4] Joaquín Pedra, and Luis Sáinz, Felipe Córcoles, Guasch, Luis. "Symmetrical and Unsymmetrical Voltage Sag Effects on Three-Phase Transformers." *Power Delivery, IEEE Transactions on* 20, no. 2 (2005): 1683-1691.

[5] Gencer, O. O., S. Ozturk, and H. Tarık Duru. "Influence of Phase-Angle Jump on Inrush Currents of Transformers." In *Electrotechnical Conference, 2006. MELECON 2006. IEEE Mediterranean*, pp. 1064-1067. IEEE, 2006.

[6] Shakarami, M. R., and A. Jalilian. "Effects of different voltage sags on three-phase transformers." In *Power Electronics, Drives and Energy Systems, 2006. PEDES'06. International Conference on*, pp. 1-5. IEEE, 2006.

[7] Masoum, Ali S., Paul S. Moses, and Mohammad AS Masoum. "Operation of three-phase three-leg power transformers subject to voltage swell." In *Electrical Power Quality and Utilisation (EPQU), 2011 11th International Conference on*, pp. 1-5. IEEE, 2011.

[8] Olguin, G., "Voltage Dip (Sag) Estimation in Power Systems based on Stochastic Assessment and Optimal Monitoring", Doctor of Philosophy dissertation. Department of Energy and Environment, Division of Electric Power Engineering, *Chalmers University of Technology*, Gothenburg, Sweden.

[9] Macken, K.J.P, Bollen, M.H.J., Belmans, R.J.M. "Mitigation of Voltage Dips Through Distributed Generation Systems", *IEEE Transaction on Industry Applications*, Vol. 40, No. 6, November/December, 2004.