A New Method for Ranking Fuzzy Numbers with Using TRD Distance Based on mean and standard deviation

Salim Rezvani 1*

1Department of Mathematics, Imam Khomeini Maritime University of Nowshahr, Nowshahr, Iran.
*Corresponding Author's E-mail: salim_rezvani@yahoo.com

Abstract

In this paper, we want to propose a new method for ranking trapezoidal fuzzy numbers with using TRD distance based on mean and standard deviation. In this method, used mean and standard deviation in membership function and inverse function for ranking fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

Keywords: Mean, Standard Deviation, Trapezoidal Fuzzy Numbers, TRD distance, Ranking Method.

1. Introduction

In most of cases in our life, the data obtained for decision making are only approximately known. In 1965, Zadeh [27] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [9]. Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. Most of the ranking procedures proposed so far in the literature cannot discriminate fuzzy quantities and some are counterintuitive. As fuzzy numbers are represented by possibility distributions, they may overlap with each other, and hence it is not possible to order them. Ranking fuzzy numbers were first proposed by Jain [11] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Since then,

Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some $\alpha$-levels of trapezoidal fuzzy numbers. Chen and Chen [5] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads and Parandian [15] proposed a method for ranking numbers by using distance between convex combination of upper and lower central gravity of $\alpha$-level and the origin of coordinate systems. Also Some of the interesting metric distance method Of Trapezoidal Fuzzy Number can be found in Chen [12]. Rezvani ([16],[22]) evaluated the system of ranking fuzzy numbers. Moreover, Rezvani [22] proposed a ranking trapezoidal fuzzy numbers based on apex angles.

In this paper, we want to propose a new method for ranking trapezoidal fuzzy numbers with using TRD distance based on mean and standard deviation. In this method, used mean and standard deviation in membership function and inverse function for ranking fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.
2. Preliminaries

Definition 1. Generally, a generalized fuzzy number $A$ is described as any fuzzy subset of the real line $R$, whose membership function $\mu_A$ satisfies the following conditions,

i) $\mu_A$ is a continuous mapping from $R$ to the closed interval $[0,1]$,

ii) $\mu_A(x) = 0, -\infty < x \leq a$,

iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a,b]$,

iv) $\mu_A(x) = w, b \leq x \leq c$,

v) $\mu_A(x) = R(x)$ is strictly increasing on $[c,d]$,

vi) $\mu_A(x) = 0, d \leq x < \infty$

where $0 < w \leq 1$ and $a,b,c$ and $d$ are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by

$$A = (a,b,c,d,w). \tag{1}$$

When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by

$$A = (a,b,c,d). \tag{2}$$

$A = (a,b,c,d,w)$ is a fuzzy set of the real line $R$ whose membership function $\mu_A(x)$ is defined as
Now, let two monotonic functions be
\[ L(x) = w \frac{x-a}{b-a}, \quad R(x) = w \frac{d-x}{d-c} \] (4)

Then the inverse functions of function \( L \) and \( R \) are \( L^{-1} \) and \( R^{-1} \) respectively.
\[ L^{-1}(x) = \frac{x}{w} (b-a) + a, \quad R^{-1}(x) = d - \frac{x}{w} (d-c) \] (5)

**Definition 2.** The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number \( A \)
\[ I(A) = \int_{0}^{1} [C L_A(\alpha) + (1-C) R_A(\alpha)] d \alpha \] (6)

And
\[ D(A) = \int_{0}^{1} [R_A(\alpha) - L_A(\alpha)] f(\alpha) d \alpha \] (7)

Here \( 0 \leq C \leq 1 \) denotes an “optimism/pessimism” coefficient in conducting operations on fuzzy numbers. The function \( f(\alpha) \) is nonnegative and increasing function on \([0, 1]\) with \( f(0) = 0, f(1) = 1 \) and \( \int_{0}^{1} f(\alpha) d \alpha = \frac{1}{2} \). The function \( f(\alpha) \) is also called weighting function. In actual applications, function \( f(\alpha) \) can be chosen according to the actual situation. In this article, in practical case, we assume that \( f(\alpha) = \alpha \).
Definition 3. TRD distance between the exponential fuzzy number A as following

\[ TRD(A) = \sqrt{[I(A)]^2 + [D(A)]^2} \]  

(8)

Definition 4. A trapezoidal fuzzy number \( A = (a,b,c,d) \) can be approximated as a symmetry fuzzy number \( S[\mu,\sigma] \), \( \mu \) denotes the mean of \( A \), \( \sigma \) denotes the standard deviation of \( A \), and the membership function of \( A \) is defined as follows:

\[
f_A(x) = \begin{cases} 
  \frac{x-(\mu-\sigma)}{\sigma} & \text{if} \quad \mu-\sigma \leq x \leq \mu \\
  \frac{(\mu+\sigma)-x}{\sigma} & \text{if} \quad \mu \leq x \leq \mu+\sigma 
\end{cases}
\]

(9)

Where \( \mu \) and \( \sigma \) are calculated as follows:

\[
\sigma = w \frac{2(d-a)+c-b}{4} 
\]

(10)

\[
\mu = w \frac{a+b+c+d}{4} 
\]

(11)

The inverse functions \( g^L \) and \( g^R \) of \( f^L \) and \( f^R \) respectively, are shown as follows:

\[
g^L(x) = (\mu-\sigma) + \sigma x 
\]

(12)

\[
g^R(x) = (\mu+\sigma) - \sigma x 
\]

(13)

3. Proposed Approach

We know of definition 4. that

\[
f_A(x) = \begin{cases} 
  \frac{x-(\mu-\sigma)}{\sigma} & \text{if} \quad \mu-\sigma \leq x \leq \mu \\
  \frac{(\mu+\sigma)-x}{\sigma} & \text{if} \quad \mu \leq x \leq \mu+\sigma 
\end{cases}
\]
So

\[ f^L = \frac{x - (\mu - \sigma)}{\sigma} \]

And

\[ f^R = \frac{(\mu + \sigma) - x}{\sigma} \]

**Theorem 1.** The values constitute of weighted averaged representative and weighted width, of the exponential fuzzy number \( A \) are following

\[ I(A) = \frac{C}{\sigma} [1 - 2\mu] + \frac{1}{\sigma} [\mu + \sigma - \frac{1}{2}] \]  

(14)

And

\[ D(A) = \frac{3\mu - 2}{3\sigma} \]  

(15)

**Proof:**

\[ I(A) = \int_0^1 [C_L(\alpha) + (1 - C) R_A(\alpha)] d\alpha = \int_0^1 [C \left( \frac{\alpha - (\mu - \sigma)}{\sigma} \right) + (1 - C) \left( \frac{(\mu + \sigma) - \alpha}{\sigma} \right)] d\alpha \]

\[ = C \left( \frac{1}{2\sigma} + \frac{\sigma - \mu}{\sigma} \right) + (1 - C) \left( \frac{\mu + \sigma}{\sigma} - \frac{1}{2\sigma} \right) = \frac{C}{\sigma} [1 - 2\mu] + \frac{1}{\sigma} [\mu + \sigma - \frac{1}{2}] . \]

And

\[ D(A) = \int_0^1 [R_A(\alpha) - L_A(\alpha)] f(\alpha) d\alpha = \int_0^1 \left( \frac{\mu + \alpha}{\sigma} - \frac{\alpha - (\mu - \sigma)}{\sigma} \right) f(\alpha) d\alpha \]

\[ = \int_0^1 \left( \frac{(\mu + \sigma) - \alpha}{\sigma} - \frac{\alpha - (\mu - \sigma)}{\sigma} \right) d\alpha = \frac{1}{2\sigma} + \frac{1}{3\sigma} - \frac{1}{3\sigma} + \frac{1}{2\sigma} = \frac{3\mu - 2}{3\sigma} \]

**Theorem 2.** TRD distance between the exponential fuzzy number \( A \) as following

\[ TRD(A) = \sqrt{\left[ \frac{C}{\sigma} [1 - 2\mu] + \frac{1}{\sigma} [\mu + \sigma - \frac{1}{2}] \right]^2 + \left[ \frac{3\mu - 2}{3\sigma} \right]^2} \]  

(16)
Definition 5. Let \( A_1 = (a_1, b_1, c_1, d_1; w_1) \) and \( A_2 = (a_2, b_2, c_2, d_2; w_2) \) be two fuzzy numbers, then
i) If \( TRD(A_1) < TRD(A_2) \), then \( A_1 < A_2 \),
ii) If \( TRD(A_1) > TRD(A_2) \), then \( A_1 > A_2 \),
iii) If \( TRD(A_1) \parallel TRD(A_2) \), then \( A_1 \parallel A_2 \).

3.1. Method to find the values of \( TRD(A) \) and \( TRD(B) \)

Let \( A = (a_1, b_1, c_1, d_1; w_1) \) and \( B = (a_2, b_2, c_2, d_2; w_2) \) be two fuzzy numbers, then use the following steps to find the values of \( TRD(A) \) and \( TRD(B) \)

* Step 1
Find \( \sigma_A \), \( \sigma_B \) and \( \mu_A \), \( \mu_B \) of formula (10), (11)
* Step 2
Find \( I_A \) and \( I_B \)
* Step 3
Find \( D_A \) and \( D_B \)
* Step 4
Calculation \( TRD(A) \) and \( TRD(B) \) and use definition 5., for ranking this method.

4. Results

Example 1. Let \( A = (0.2, 0.4, 0.6, 0.8; 0.35) \) and \( B = (0.1, 0.2, 0.3, 0.4; 0.7) \) be two generalized trapezoidal fuzzy number, then

* Step 1
\[ \sigma_A = 0.12, \sigma_B = 0.12 \]

And
\[
\mu_A = 0.17, \quad \mu_B = 0.17
\]

* Step 2
\[
I_A = 0.08C - 1.75, \quad I_B = 0.08C - 1.75
\]

* Step 3
\[
D_A = -4.14, \quad D_B = -4.14
\]

* Step 4
\[
TRD(A) = \sqrt{(0.08C - 1.75)^2 + (-4.14)^2}
\]

For
\[
C = 0 \Rightarrow TRD(A) = 4.5,
\]
\[
C = 0.5 \Rightarrow TRD(A) = 4.48,
\]
\[
C = 1 \Rightarrow TRD(A) = 4.46,
\]
\[
TRD(B) = \sqrt{(0.08C - 1.75)^2 + (-4.14)^2}
\]
\[
C = 0 \Rightarrow TRD(B) = 4.5,
\]
\[
C = 0.5 \Rightarrow TRD(B) = 4.48,
\]
\[
C = 1 \Rightarrow TRD(B) = 4.46,
\]

Then \( TRD(A) \cap TRD(B) \Rightarrow A \sqcap B \).

**Example 2.** Let \( A = (0.1,0.2,0.4,0.5;1) \) and \( B = (0.1,0.3,0.3,0.5;1) \) be two generalized trapezoidal fuzzy number, then

* Step 1
\[
\sigma_A = 0.25, \quad \sigma_B = 0.2
\]

And
\[
\mu_A = 0.3, \quad \mu_B = 0.3
\]

* Step 2
\[ I_A = 1.6C + 0.2, \quad I_B = 1.6C + 0.25 \]

* Step 3

\[ D_A = -1.47, \quad D_B = -1.83 \]

* Step 4

\[ TRD(A) = \sqrt{(1.6C + 0.2)^2 + (-1.47)^2} \]

For
\[ C = 0 \Rightarrow TRD(A) = 1.48, \]
\[ C = 0.5 \Rightarrow TRD(A) = 1.78, \]
\[ C = 1 \Rightarrow TRD(A) = 2.32, \]

\[ TRD(B) = \sqrt{(1.6C + 0.25)^2 + (-1.83)^2} \]

\[ C = 0 \Rightarrow TRD(B) = 1.85, \]
\[ C = 0.5 \Rightarrow TRD(B) = 2.11, \]
\[ C = 1 \Rightarrow TRD(B) = 2.6, \]

Then, \( TRD(A) < TRD(B) \Rightarrow A < B \).

**Example 3.** Let \( A = (0.1, 0.2, 0.4, 0.5; 1) \) and \( B = (1, 1, 1, 1; 1) \) be two generalized trapezoidal fuzzy number, then

* Step 1

\[ \sigma_A = 0.25, \quad \sigma_B = 0 \]

And

\[ \mu_A = 0.3, \quad \mu_B = 1 \]

* Step 2

\[ I_A = 1.6C + 0.2, \quad I_B = \infty \]

* Step 3
\[ D_A = -1.47 , D_B = \infty \]

* Step 4

\[ TRD(A) = \sqrt{[1.6C + 0.2]^2 + [-1.47]^2} \]

For

\[ C = 0 \Rightarrow TRD(A) = 1.48 , \]
\[ C = 0.5 \Rightarrow TRD(A) = 1.78 , \]
\[ C = 1 \Rightarrow TRD(A) = 2.32 , \]

\[ TRD(B) = \infty \]

\[ C = 0 \Rightarrow TRD(B) = \infty , \]
\[ C = 0.5 \Rightarrow TRD(B) = \infty , \]
\[ C = 1 \Rightarrow TRD(B) = \infty , \]

Then, \( TRD(A) < TRD(B) \Rightarrow A < B \).

**Example 4.** Let \( A = (-0.5, -0.3, -0.3, -0.1; 1) \) and \( B = (0.1, 0.3, 0.3, 0.5; 1) \) be two generalized trapezoidal fuzzy number, then

* Step 1

\[ \sigma_A = 0.1 , \sigma_B = 0.2 \]

And

\[ \mu_A = -0.3 , \mu_B = 0.3 \]

* Step 2

\[ I_A = 16C - 7 , I_B = 1.6C + 0.25 \]

* Step 3

\[ D_A = -9.7 , D_B = -1.83 \]

* Step 4
\[ TRD(A) = \sqrt{(16C - 7)^2 + [-9.7]^2} \]

For
\[
C = 0 \Rightarrow TRD(A) = 11.9,
C = 0.5 \Rightarrow TRD(A) = 9.7,
C = 1 \Rightarrow TRD(A) = 13.23,
\]
\[ TRD(B) = \sqrt{(1.6C + 0.25)^2 + [-1.83]^2} \]
\[
C = 0 \Rightarrow TRD(B) = 1.85,
C = 0.5 \Rightarrow TRD(B) = 2.11,
C = 1 \Rightarrow TRD(B) = 2.6,
\]
Then, \( TRD(A) > TRD(B) \Rightarrow A > B \).

**Example 5.** Let \( A = (0.3, 0.5, 0.5, 1; 1) \) and \( B = (0.1, 0.6, 0.6, 0.8; 1) \) be two generalized trapezoidal fuzzy number, then

* Step 1

\[
\sigma_A = 0.35, \sigma_B = 0.17
\]

And
\[
\mu_A = 0.57, \mu_B = 0.52
\]

* Step 2

\[
I_A = -0.4C + 1.2, I_B = -0.23C + 1.12
\]

* Step 3

\[
D_A = -0.28, D_B = -0.86
\]

* Step 4

\[
TRD(A) = \sqrt{(-0.4C + 1.2)^2 + [-0.28]^2}
\]

For
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\[ C = 0 \Rightarrow TRD(A) = 1.23, \]
\[ C = 0.5 \Rightarrow TRD(A) = 1.04, \]
\[ C = 1 \Rightarrow TRD(A) = 0.85, \]

\[ TRD(B) = \sqrt{[-0.23C + 1.12]^2 + [-0.86]^2} \]

\[ C = 0 \Rightarrow TRD(B) = 1.41, \]
\[ C = 0.5 \Rightarrow TRD(B) = 1.32, \]
\[ C = 1 \Rightarrow TRD(B) = 1.24, \]

Then, \( TRD(A) < TRD(B) \Rightarrow A < B \).

**Example 6**  
Let \( A = (0, 0.4, 0.6, 0.8; 1) \) and \( B = (0.2, 0.5, 0.5, 0.9; 1) \) and \( C = (0.1, 0.6, 0.7, 0.8; 1) \) be two generalized trapezoidal fuzzy number, then

* Step 1  
  \[ \sigma_A = 0.45, \sigma_B = 0.35, \sigma_C = 0.37 \]

And

\[ \mu_A = 0.45, \mu_B = 0.52, \mu_C = 0.55 \]

* Step 2  
  \[ I_A = 0.22C + 0.9, I_B = -0.11C + 1.06, I_C = -0.27C + 1.13 \]

* Step 3  
  \[ D_A = -0.48, D_B = -0.42, D_C = -0.31 \]

* Step 4  
  \[ TRD(A) = \sqrt{(0.22C + 0.9)^2 + [-0.48]^2} \]

For

\[ C = 0 \Rightarrow TRD(A) = 1.02, \]
\[ C = 0.5 \Rightarrow TRD(A) = 1.12, \]
Then, $TRD(A) > TRD(B) > TRD(C) \Rightarrow A > B > C$.

**Example 7.** Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (-2, 0, 0, 2; 1)$ be two generalized trapezoidal fuzzy number, then

* Step 1

$\sigma_A = 0.25, \ \sigma_B = 2$

And

$\mu_A = 0.3, \ \mu_B = 0$

* Step 2

$I_A = 1.6C + 0.2, \ I_B = 0.5C + 0.75$

* Step 3

$D_A = -1.47, \ D_B = -0.33$

* Step 4

$$TRD(A) = \sqrt{(1.6C + 0.2)^2 + (-1.47)^2}$$
For

\[ C = 0 \Rightarrow TRD(A) = 1.48, \]
\[ C = 0.5 \Rightarrow TRD(A) = 1.78, \]
\[ C = 1 \Rightarrow TRD(A) = 2.32, \]

\[ TRD(B) = \sqrt{(0.5C + 0.75)^2 + (-0.33)^2} \]
\[ C = 0 \Rightarrow TRD(B) = 0.82, \]
\[ C = 0.5 \Rightarrow TRD(B) = 1.05, \]
\[ C = 1 \Rightarrow TRD(B) = 1.29, \]

Then, \( TRD(A) > TRD(B) \Rightarrow A > B \).

It is clear from Table 1, the results of the proposed approach are same as obtained by using the existing approach.

**Table 1: A comparison of the ranking results for different approaches**

<table>
<thead>
<tr>
<th>Approaches</th>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
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<tbody>
<tr>
<td>Proposed approach</td>
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<td>A&lt;B</td>
<td>A&lt;B</td>
<td>A&gt;B</td>
<td>A&lt;B</td>
<td>A&gt;B&gt;C</td>
<td>A&gt;B</td>
</tr>
</tbody>
</table>
Conclusion

In this paper, we want to propose a new method for ranking trapezoidal fuzzy numbers with using TRD distance based on mean and standard deviation. The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

References


