Optimum Design of a SRM Using FEM & PSO

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Abstract

Nowadays the use of the Switched Reluctance Motors (SRMs) has been increased in various home and industrial applications considerably. Despite of many advantages of this type of motors such as simple structure, low cost, and high reliability, the main disadvantage of them is their high torque pulsation. This paper presents a novel method to optimize a typical SRM such that the torque ripple reaches its minimum value. Meanwhile, the torque average and the motor efficiency become maximum. It is shown that the pole width to the pole pitch ratio have a great impact on the torque ripple and torque average for both stator and rotor poles. Finite Element Method (FEM) was used to obtain the torque ripple, the torque average and the motor efficiency for a large number of ratios. A functional relationship was developed between the input and the output parameters. Then, the Particle Swarm Optimization (PSO) was used to find the optimum design.

Keywords: Finite element, Particle Swarm Optimization, Switched reluctance motors.

1. Introduction

Switched reluctance motors are in the family of reluctant motors which have very simple structure [1], [2]. This motor has salient poles in the rotor and stator and has coils only on the stator poles. The advantages of these types of motors can be listed as: simplicity of structure, low price of manufacturing, variability of speed, safe power controlling, high reliability, long lifespan, high ratio of torque to volume, and the resistance against overheating [3], [4]. These advantages have caused a widespread usage of such motors in the industry. Despite of these advantages, switched reluctance motors have some disadvantages too. In these motors the stator currents waveforms are complex and also they have torque ripples that cause fuss and acoustic noise [5][6][7][8]. The cause of these ripples in such motors, at first could be attributed to the working in the saturation and also to discontinuous mechanism of the torque production. There are two general methods for decreasing the ripples of the torque. The first one is changing the design of the motor from a magnetic point of view [9][10][11][12][13][14]. And the second one, is changing the electronic control of the motor-driven [15][16][17]. The first method is presented in this paper. Considering the conducted studies on this area, it has been observed that such studies are in two general categories. In the first state, there is no fundamental change in the motor structure, instead, the influence of the motor dimensions on the cogging torque were inspected; parameters such as stator pole arch, motor axial length, slot skewing, the ratio of the external to the internal radius and so on. In the second one, a new structure has been suggested for such motors. As a sample, it has been worked on the stator and rotor poles [18]. At first, the angle of the stator and rotor poles have been changed from 15° to 23° and 15° to 30° respectively. Then, the motor torque and its ripple are obtained using the Finite...
Element Method (FEM) in the static mode. Finally, using these values, an Artificial Neural Network (ANN) has been trained for optimizing the dimensions of the stator and the rotor poles in order to have more average torque with less torque ripple. A new structure, including two rotors, has been suggested for the motor in [19]. It has been observed that the average torque increased and the torque ripple decreased. In [20], it has been worked on the stator structure. In order to have less saturation, some changes are applied to the edge of the stator teeth. It was observed that the torque and its ripple reached a better condition.

This paper focuses on the ratio of the width of the teeth to the pitch of the pole. At first, a motor with defined dimensions has been considered. Changing the ratio of the tooth width to the pole pitch for the stator and rotor, and simulating motor by FEM, a lot of data have been obtained that has been arranged in a table. In order to gain the points which are not available in the table of the data, the interpolation method has been used. Then, using the Particle Swarm Optimization (PSO), an optimum value has been achieved for the ratio of the pole width to the pole pitch for both stator and rotor. The cost function has been defined so that the motor would have the maximum average torque, minimum torque ripple and maximum efficiency.

2. The Equation of the Torque

The equation of the voltage for one phase of the switched reluctance motor is as follow:

\[ v = r i + \frac{d\lambda}{dt} \]  \hspace{1cm} (1)

Here, \( v \) is the voltage of the phase, \( r \) is the resistance of the coil and \( \lambda \) is the flux linkage. Supposing that the core is linear and the rotor rotates with the angular speed of \( \omega \), the voltage equation will be as follow:

\[ v = r i + L \frac{di}{dt} + i \omega \frac{dL(\theta)}{d\theta} \]  \hspace{1cm} (2)

Here, \( L \) is inductance and \( \theta \) is the position of the rotor. From equation (2), the equation of the power in motor will be as follows:

\[ vi = ri^2 + Li \frac{di}{dt} + i^2 \omega \frac{dL(\theta)}{d\theta} \]  \hspace{1cm} (3)

\[ vi = ri^2 + \frac{d}{dt} \left( \frac{1}{2} Li^2 \right) + \frac{1}{2} i^2 \omega \frac{dL(\theta)}{d\theta} \]  \hspace{1cm} (4)

The left side of the equation (4) is the input power. The first term in the right hand side of the equation is the resistive power losses. The second term is the power which is stored in the magnetic field, and the third part is the mechanical power. So, the torque equation can be achieved as:

\[ \tau = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta} \]  \hspace{1cm} (5)

Figure.1 shows a typical torque waveform produced by such motors. This waveform has been obtained from the finite element analysis. The figure shows a lot of ripple in the torque waveform.

3. The Torque Ripple

As it is clear in Figure.1, the torque is changing from the minimum to the maximum value. In various papers, different formulas have been used for measuring the ripples of the torque. In this paper, the ripple of the torque is defined as follows:
\[ \Delta \tau = \frac{\tau_{\text{max}} - \tau_{\text{ave}}}{\tau_{\text{max}}} \]  

Here, \( \Delta \tau \) is the ripple of the torque, \( \tau_{\text{max}} \) is the maximum torque, and \( \tau_{\text{ave}} \) is the average torque.

4. Finite Element Analysis

The governing equation for 2D magneto static problem can be represented as follow:

\[ \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu J_s \]  

Where, \( \mu \) is magnetic permeability, \( A \) is the vector magnetic potential and \( J_s \) is the current density in the coil area. The above differential equation is converted to algebraic equation through the Galerkin method:

\[ [S]_{n \times n} [A]_{n \times 1} = [D]_{n \times 1} \]  

In which, \([S]\) is the coefficient matrix, \([A]\) is the vector of potentials, \([D]\) is a fixed vector and \(n\) is the number of the nodes. Matrix \([S]\) and vector \([D]\) are constant and they depend on the geometry of the 2D space and the way of meshing [20]. The voltage equation for one phase is as follows:

\[ v_k = r_k i_k + \frac{d\lambda_k}{dt} \]  

Where, \(v_k\) is the phase voltage source, \(i_k\) is the phase current, \(r_k\) is the resistance of the phase and \(\lambda_k\) is the flux linkage of the phase winding. The flux linkage, \(\lambda_k\), is achievable through the following equation:

\[ \lambda_k = l_s [D]^T [A] \]  

In that \(l_s\) is the axial length. The phase current, \(i_k\), is in relationship with the current density in the coil area, \(J_s\), as follow:

\[ i_k = \frac{s_j}{n_w} J_s \]
where, $S_j$ is the coil’s cross-sectional area and $n_w$ is the number of turns in that area. Equations (8) to (11) have been indicated in the form of matrix as follow:

$$
\begin{bmatrix}
    [S]
    [D]
\end{bmatrix}^T
-\frac{s_j \, r \, \Delta t}{n_w \, I_z}
\begin{bmatrix}
    [A(t + \Delta t)]
    \\
    -[J_z(t + \Delta t)]
\end{bmatrix}
=
\begin{bmatrix}
    0_{(s \times n)}
    [A(t)]
\end{bmatrix}
+
\begin{bmatrix}
    0_{n \times 1}
\end{bmatrix}
\begin{bmatrix}
    \frac{\Delta t}{I_z}
\end{bmatrix}
\begin{bmatrix}
    0_{n \times 1}
\end{bmatrix}
\begin{bmatrix}
    v_z(t + \Delta t)
\end{bmatrix}
$$

(12)

After solving the equation (12) and determining magnetic potential, the flux density can be achieved from:

$$
\bar{B} = \nabla \times \bar{A}
$$

(13)

where, $\bar{B}$ is the flux density vector and $\bar{A}$ is the magnetic potential vector. According to Maxwell stress formulation, the electromagnetic torque can be represented as follow [21].

$$
\tau = \frac{I_z}{\mu_0} \int_0^{2\pi} r^2 B_r B_\theta d\theta
$$

(14)

where, $B_r$ and $B_\theta$ are the radial and tangential components of vector $\bar{B}$ respectively and $\tau$ is the motor torque.

5. Determining the Motor Efficiency

Efficiency is one of the important factors which is affected by the design of the motor. To achieve efficiency of a motor, the below equation could be used:

$$
\eta = \frac{P_{out}}{P_{in}} \times 100
$$

(15)

In this equation, $\eta$ is efficiency, $P_{in}$ is the input power and $P_{out}$ is the output power of the motor. To gain the input power, equation (16) must be used.

$$
P_{in} = \frac{1}{T} \int_0^T v_i \, dt
$$

(16)

And also the output power is achieved through the below equation.

$$
P_{out} = \tau_{ave} \omega
$$

(17)

6. Particle Swarm Optimization (PSO)

Particle Swarm Optimization was designed by Kennedy and Eberhart for the first time [22]. The primary idea of this algorithm has been deduced from the flight of the birds. In the PSO algorithm, there are some elements known as particles, which are expanded in the search-space. Each particle can be an N-dimensional vector. The goal is to find the best particle (or best position) which has the minimum value of the objective function. At first, the particles are initialized with random values. Then, the objective function is calculated for each particle. After that, from current position and velocity of the particle and the best position of it (from beginning up to now) and the position of the best particle, the next position and the next velocity of each particle can be found out. This process is done for all particles in each iteration. After some iterations (depends on the required accuracy), the
particle which has the minimum value of the objective function is available. The following equations indicate updating process of the velocity and position for each particle [23].

\[
v_{i+1}(p) = w_i v_i(p) + c_i rand_i (\text{best}_i(p) - x_i(p)) + d_i rand_i (\text{best}_p - x_i(p))
\]

\[
x_{i+1}(p) = x_i(p) + v_{i+1}(p)
\]

Where, \(v_i(p)\) and \(x_i(p)\) are the velocity and the position of each particle respectively, \(c_i\) is the local learning coefficient and \(d_i\) is the global learning coefficient. They are in the range of 0 to 2 and they affect on the convergent speed [23]. The inertia weight, \(w\), is very important in the convergence of PSO. Increasing \(w\) increases the convergence speed and it means less exploration property. To choose a suitable \(w\), the following equation can be used [24]:

\[
w(k) = w_{max} - \left(\frac{w_{max} - w_{min}}{iter_{max}}\right) \times iter
\]

Here, \(iter_{max}\) is the maximum number of iterations, \(iter\) is the number of iteration, \(w_{max}\) is the primary weight, and \(w_{min}\) is the final weight. The primary value for \(w\) is around 1 and it will be decreased towards zero gradually.

7. Selecting the Appropriate Parameters for the Optimization

Four different parameters are defined in the SRM. These four parameters are:

\[
\beta_s = \frac{1}{\alpha_s}, \beta_r = \frac{1}{\alpha_r}, \frac{R_{ir}}{P_r} = R_{sr}, \frac{R_{or}}{P_s} = R_{sr}
\]

where \(\alpha_s\) is the stator pole pitch, \(\beta_s\) is the stator pole width, \(\alpha_r\) is the rotor pole pitch and \(\beta_r\) is the rotor pole width and are shown in Figure.5. Moreover \(R_{ir}\) is the rotor inner radius, \(R_{or}\) is the rotor outer radius, \(R_{is}\) is the stator inner radius and \(R_{os}\) is the stator outer radius. These four parameters, \(\gamma_s\), \(\gamma_r\), \(P_s\) and \(P_r\), were changed in the range of 0.35 to 0.65. The other specifications of the motor are listed in Table.1. The average torque and the torque ripple were calculated at each point. Results are shown in Figure.2 and Figure.3 respectively. It is seen that the role of \(\gamma_s\) and \(\gamma_r\) on the average torque and the torque ripple is much higher than the \(P_r\) and \(P_s\). So \(\gamma_s\) and \(\gamma_r\) were chosen as the optimization parameters.

![Figure 2: The changes of the average torque](image-url)
8. The Optimization of the Studied Motor

The parameters of the studied switched reluctance motor are listed in Table 1. This motor has been modeled by the finite element software (Maxwell-13). Figure 4 shows the contour-plot of the magnetic flux in a typical condition of the motor analysis. As mentioned before, $\gamma_s$ and $\gamma_r$ are the important factors in the optimization process. $\gamma_s$ was changed in the range of 0.53 to 0.73 and $\gamma_r$ was changed in the range of 0.33 to 0.5. The step of the changes was considered to be 0.01. In each case, the average torque, the torque ripple and the efficiency are calculated using finite element analysis. In this way 504 output data were obtained. Some of them are shown in Table 2 to Table 4. The graph of the results is shown in Figure 6 to Figure 8.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of stator poles</td>
<td>12</td>
</tr>
<tr>
<td>Number of rotor poles</td>
<td>8</td>
</tr>
<tr>
<td>Number of phases</td>
<td>3</td>
</tr>
<tr>
<td>Stator pole pitch</td>
<td>30°</td>
</tr>
<tr>
<td>Rotor pole pitch</td>
<td>45°</td>
</tr>
<tr>
<td>Stator pole arc</td>
<td>15°</td>
</tr>
<tr>
<td>Rotor pole arc</td>
<td>22.5°</td>
</tr>
<tr>
<td>External diameter of the stator</td>
<td>140 mm</td>
</tr>
<tr>
<td>Internal diameter of the stator</td>
<td>84 mm</td>
</tr>
<tr>
<td>Depth of the stator slot</td>
<td>14.46 mm</td>
</tr>
<tr>
<td>Minimum air gap</td>
<td>1 mm</td>
</tr>
<tr>
<td>External diameter of the rotor</td>
<td>83 mm</td>
</tr>
<tr>
<td>Internal diameter of the rotor</td>
<td>49.8 mm</td>
</tr>
<tr>
<td>Motor length</td>
<td>65 mm</td>
</tr>
<tr>
<td>Winding turns</td>
<td>142</td>
</tr>
</tbody>
</table>
Figure 4: Flux lines of the switch reluctance motor due to finite element analysis using Maxwell-13 software.

Figure 5: Representation of the pole pitch and slot width for both stator and rotor.

Figure 6: The changes of average torque in the range of $0.53 \leq \gamma_s \leq 0.73$ and $0.33 \leq \gamma_r \leq 0.53$.

Figure 7: The changes of the torque ripple in the range of $0.53 \leq \gamma_s \leq 0.73$ and $0.33 \leq \gamma_r \leq 0.53$. 
Using finite element analysis, a discrete space with the high accuracy points is available. With an acceptable accuracy, it is possible to obtain a continuous space of $\gamma_s$, $\gamma_r$ and related torque or efficiency, by means of linear interpolation. The final goal is to find the optimum $\gamma_s$ and $\gamma_r$ with the maximum average torque and efficiency and the minimum torque ripple. It can be done using the PSO. The following equation is defined as objective function and can be used for optimization of $\gamma_s$ and $\gamma_r$.

$$OPT = \left( \frac{\Delta \tau}{\tau_{\text{max}}} \right) - \left( \frac{\eta}{\eta_{\text{max}}} \right) - \left( \frac{\tau_{\text{ave}}}{\tau_{\text{ave,max}}} \right)$$

(22)
The PSO was converged in 13 iterations, as it is clear in Figure 9. The objective function’s variations are shown in Figure 10. The optimal values for $\gamma_s$ is 0.55 and for $\gamma_r$ is 0.53. In this case the average torque is 6.45, the ripple torque is 0.54, and the efficiency is 80.63%. These values are the best choice for $\gamma_s$ and $\gamma_r$.

![Figure 9: Objective function within each PSO iteration](image1)

![Figure 10: The minimum point of the objective function](image2)

**CONCLUSION**

According to the widespread usage of the Switch Reluctance Motors in home and industrial applications, study of these motors is now became a great concern. One of the problems of this motor is the torque ripple which causes noise. In this paper it is shown that the most effective factors on the torque ripple are the ratio of the pole width to the pole pitch for stator and rotor ($\gamma_s$ and $\gamma_r$). On the other hand, decreasing the torque ripple in some cases, may cause a decreasing in the average torque. Therefore, each optimization procedure must obtain the minimum of the ripple with the maximum of the average. Moreover, the motor efficiency is important too, therefore, the optimization method must be performed in a way that it reaches its maximum value. To achieve this multi-purpose optimization, the PSO has been used. At first, values of the average torque, torque ripple, and the efficiency have been gained for a lot of $\gamma_s$ and $\gamma_r$, using the finite element software. By means of interpolation, average torque, torque ripple, and efficiency have been represented as continuous functions of $\gamma_s$ and $\gamma_r$. Finally, using PSO, the best value of $\gamma_s$ and $\gamma_r$ has been achieved. The conducted analysis by finite element method, validates the achieved optimum points.

**References**


