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Trajectory Estimation for Ballistic Missile in Free-Flight Phase using Extended Kalman Filter based on Radar Measurments

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Abstract

It is possible to respond to threats of ballistic missile by using radar measurements. However, these measurments from radar are mainly used to detect ballistic missile, not to track the object with precision due to noise. In this paper, an Extended Kalman Filter trajectory tracking algorithm for the ballistic missile using radar measuremenst is proposed in the free flight phase. The suggested algorithm estimates the trajectory of the ballistic missile by solving the optimization problem. This study, re-examine some of the often neglected practical constraints of radar tracking and motion modeling. The filter is described for obtaining sets of non-linear state equations such as position and velocity whose acceleration is perturbed due to random factors, used to compute covariances required for measurement update. The proposed methodology ensures guaranteed convergence of the tracking filter, what is more, the estimation precision is high.

Keywords: *ballistic, missile, EKF, trajectory, radar, estimation.*

1. Introduction

In the early 1950s, filtering estimation method had been appreciated highly in the ballistic missile detection and defensive system [8]. The countermeasures that destroys ballistic missile (BM) before they reach their target, is being developed in many countries around the world, including the United States. In order to effectively react to the BM threats, it is important to rapidly detect and accurately track the trajectory. Various radars such as an early warning radar and a detection radar have been used for these reasons, and studies have been carried out to track this kind of targets. However, detection and tracking are not possible with radar until the BM(s) rise above the horizon because of the curvature of the earth, thereby reducing response time [4]. One vital step in BM tracking is, position and velocity estimation for interception purposes which is provided to a missile defense system [7]. The desired information on such threats consists of its azimuth, elevation, range, heading, and speed. Target tracking could be performed in the coordinates specified by these measurements in order to obtain reliable estimates of position and velocity errors for the circumstances under investigation [3]. Tracking of BM can be done with the help of a ground-based radar but traced parameters contain certain

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uncertainty and error. This error may be decreased by a nonlinear version of Kalman Filter (KF), that uses a collection of equations and consecutive information input (from radar) to estimate the accurate value. However, this filter is simply appropriate for linear system model and thence, it cannot be used to trace the motion of BM in re-entry section that is extremely nonlinear and unpredictable due to the parameters like drag, drift, ballistic constant, and atmospheric condition. Just in case of a non-linear method, EKF adopts a way refers to as Taylor series growth to linearize the non-linear state model[3].

2. Problem Formulation

The equation of motion of BM is more complicated in the spherical coordinate system, so in order to simplify the tracking processes, it is usually best to be described in cartesian coordinate, therefore the tracking must involve the following; coordinate system, earth model, force model, motion model, and filtering algorithm.

3. Extended Kalman Filter

Kalman filter (KF) is designed to estimate the states of a linear system based on measurements that are a linear function of the states. However, in real life situation, either the system model is a non-linear function of the states and/or the measurements are non-linear functions of the states. Due to the attractiveness of the KF, designers have developed a new set of mathematics the KF theory which results to Extend Kalman Filter (EKF) for non-linear situations[5].

3.1 EKF algorithm

The equation of states for non-linear system is

$$\boldsymbol{X}(k+1) = f(\boldsymbol{X}(k)) + w(k) \tag{1}$$

The state Jacobi is given by $\boldsymbol{F}(k) = \left.\frac{\partial f(\boldsymbol{X}(k))}{\partial \boldsymbol{X}(k)}\right|_{\boldsymbol{X}(k)=\hat{\boldsymbol{X}}(k|k)}$

w(k) is the process noise

the measurement equation is given by

$$\boldsymbol{Z}(k) = h(\boldsymbol{X}(k)) + v(k) \tag{2}$$

The measurement Jacobi is given by $\boldsymbol{H}(k) = \left.\frac{\partial \mathrm{h}(\boldsymbol{X}(k))}{\partial \boldsymbol{X}(k)}\right|_{\boldsymbol{X}(k)=\hat{\boldsymbol{X}}(k+1|k)}$

Where *v(k) is the* measurement noise

prediction of state: $\hat{\boldsymbol{X}}(k+1|k) = f(k, \hat{\boldsymbol{X}}(k|k))$

prediction covariance: $\boldsymbol{P}(k+1|k) = \boldsymbol{F}(k) \cdot \boldsymbol{P}(k|k) \cdot \boldsymbol{F}'(k) + \boldsymbol{Q}(k)$

Measurement prediction: $\hat{\boldsymbol{Z}}(k+1|k) = h(k+1, \hat{\boldsymbol{X}}(k+1|k))$ (3)

Kalman gain is given by:

$$\boldsymbol{K} = \boldsymbol{P}(k+1|k)\boldsymbol{H}'(k)\left[\boldsymbol{H}(k)\boldsymbol{P}(k+1|k)\boldsymbol{H}'(k) + \boldsymbol{R}(k+1)\right]^{-1}$$

Status update:

$$\hat{\boldsymbol{X}}(k+1|k+1) = \hat{\boldsymbol{X}}(k+1|k) + \boldsymbol{K}[\boldsymbol{Z}(k+1) - \hat{\boldsymbol{Z}}(k+1|k)]$$

Covariance update: $\boldsymbol{P}(k+1|k+1) = [\boldsymbol{I} - \boldsymbol{KH}(k)]\boldsymbol{P}(k+1|k)$ (4)

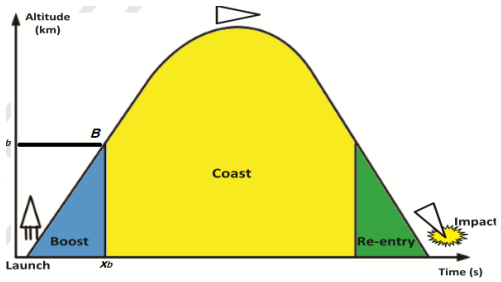


Figure 1: Ballistic flight phases[7].

4. Tracking coordinate system

It is convenient to adopt the ECEF (Earth-centered Earth-fixed) coordinate system [1]. The radar station is based on the ENU (East North Up) coordinate system. as shown in fig. 2.

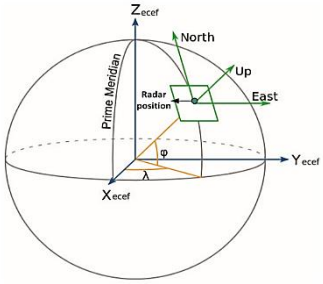


Figure 2: The illustration of the global ECEF and local ENU coordinate systems. Note, the x and y axes of the ECEF coordinate system lie in the equatorial plane while the x axis pointing towards the Greenwich. And the z-axis of the ECEF coordinate system is the conventional terrestrial pole (CTP) axis. With Φ and λ being the latitude and longitude of the radar [1].

5. System Dynamics

This section presents the state modeling system used for the BM tracking. The flight of a ballistic object consists of three phases[1] shown below in fig. 1.

We are going to make much emphases on the free-flight phase for this present investigation.

We made the following assumptions which is similar to [6],

- Earth is perfectly spherical.
- The effect of the aerodynamic lift is currently neglected as in [1].

The state equations describing the target motion are the following:

$$\dot{p} = v \quad (5)$$

$$\dot{v} = a = a_D + a_G \quad (6)$$

where; a is the total acceleration acting on the target has been decomposed into two terms, explained below;

5.1 Drag acceleration (a_D)

is opposite to target velocity according to [6] which is given by

$$a_D \approx \frac{\rho(h(t))v^2(t)}{2\beta} \tag{7}$$

where $v(t) = \sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)}$ is the magnitude of the target's velocity at time *t*; β represents ballistic coefficient, $h(t) = \sqrt{p_x^2(t) + p_y^2(t) + p_z^2(t)} - R_I$ is the target's altitude above the sea level at time (*t)* and R_T = 6,378,173(m) being the Earth's radius.

$\rho(h)$ represents the air density as a function of altitude *(h)* given by $\rho(h) = \rho_0 e^{-kh}$, where k=0.00014141 m^{-1} and ρ_0 =1.22kg/ m^3 [1], [6].

5.2 Gravitational acceleration (a_G)

The points from the target to the earth's center and its magnitude is given by Newton's law of universal gravitation as [6]

$$a_G = \frac{\mu_G}{p^2(t)} \tag{8}$$

Where $\mu_G = 3.985325x10^{14}\ Nm^2kg^{-1}$ and The distance of the target from the Earth's center at time (t) is

$$\left|p(t)\right| = \sqrt{p_x^2(t) + p_y^2(t) + p_z^2(t)} \tag{9}$$

Finally gathering the cartesian components of the drag and gravitational accelerations by summing up (7 and 8), The state equations describing the target's motion in the free flight phase, are:

$$\left.\begin{aligned} \dot{p}_x &= v_x \\ \dot{p}_y &= v_y \\ \dot{p}_z &= v_z \\ a_x = \dot{v}_x &= -\frac{p(h)}{2\beta} VV_x - \frac{\mu_G p_x}{p^3} \\ a_y = \dot{v}_y &= -\frac{p(h)}{2\beta} VV_x - \frac{\mu_G p_x}{p^3} \\ a_x = \dot{v}_z &= -\frac{p(h)}{2\beta} VV_x - \frac{\mu_G p_x}{p^3} \end{aligned}\right\} \tag{10}$$

Notice that in equation (10) the drag acceleration term vanishes exponentially with increasing altitude (*h)* and, therefore, these equations describe the free-flight segment.

6. The State Model

In order to use EKF, the nonlinear system of equations in the state model must be linearized and dispersed to the state equation below [7].

$$X_{k+1} = F_k X_k + w_k \tag{11}$$

where;

X_{k+1} is the current state and X_k is the previous state

The state parameter is given by $X_k = \begin{bmatrix} p_x & p_y & p_z & v_x & v_y & v_z & n & q \end{bmatrix}_k^T$ (12)

The equation of motion is given by

$$\dot{X}_k = \begin{bmatrix} v_x \\ v_y \\ v_z \\ a_x \\ a_y \\ a_z \end{bmatrix}_k \tag{13}$$

6.1 Free-flight segment state function

$$\begin{bmatrix} v \\ a \end{bmatrix}_{k+1}^{ff} = F_k^{ff} \begin{bmatrix} v \\ a \end{bmatrix}_k^{ff} + G_k{}^{ff} w^{ff}{}_k \tag{14}$$

$$F_k^{ff} = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ F_{4,1}.T & F_{4,2}.T & F_{4,3}.T & F_{4,4}.T & F_{4,5}.T & F_{4,6}.T \\ F_{5,1}.T & F_{5,2}.T & F_{5,3}.T & F_{5,4}.T & F_{5,5}.T & F_{5,6}.T \\ F_{6,1}.T & F_{6,2}.T & F_{6,3}.T & F_{6,4}.T & F_{6,5}.T & F_{6,6}.T \end{bmatrix} \tag{15}$$

$$w^{ff}{}_k = \begin{bmatrix} w^{x,ff}{}_k w^{y,ff}{}_k w^{z,ff}{}_k w^{v_x,ff}{}_k w^{v_y,ff}{}_k w^{v_z,ff}{}_k \end{bmatrix}^T \tag{16}$$

Note: Refer to the Appendix for the Jacobi *(F)*

Equation (16) represents the free-flight model acceleration uncertainties.

$$G_k{}^{ff} = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

The process noise covariance is given by $w_k \sim N(0, Q)$ therefore;

$$Q^{ff} = diag\{\sigma_{p_x}^2, \sigma_{p_y}^2, \sigma_{p_z}^2, \sigma_{v_x}^2, \sigma_{v_y}^2, \sigma_{v_z}^2, \} \tag{18}$$

7.1 The Measurement Model

The BM is observed from a radar station which measures (with a sampling time T) the spherical coordinates (i.e., range r_r, azimuth b_r, and elevation e_r) of the target with respect to the reference system.

Measurements are described by a position vector in this coordinate and the dynamics of BM is performed in Cartesian coordinates.

The non-linear measurement equation is given by,

$$Z_k = h_k(X_k) + v_k \tag{19}$$

where $h_k = [r_r \quad b_r \quad e_r]^T$ (20)

$$\begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} = \begin{bmatrix} r_r \cos b_r \cos e_r \\ r_r \sin b_r \cos e_r \\ r_r \sin e_r \end{bmatrix} \tag{21}$$

where

$$r_r = \sqrt{x_k^2 + y_k^2 + z_k^2} \tag{22}$$

$$b_r = \tan^{-1}\left(\frac{x_k}{y_k}\right) \tag{23}$$

$$e_r = \tan^{-1}\left(\frac{z_k}{x_k + y_k}\right) \tag{24}$$

The measurement noises (v_k) which is a 3x1 vector.

$v_k \sim N(0, \boldsymbol{R})$ v_k is assumed to be white Gaussian noise with zero mean with the following standard deviations; range measurement errors (σ_r), azimuth error (σ_b) and elevation error(σ_e)[8].

$$\boldsymbol{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_b^2 & 0 \\ 0 & 0 & \sigma_e^2 \end{bmatrix} \boldsymbol{A}^T \tag{25}$$

Where $$\boldsymbol{A} = \begin{bmatrix} \cos b \cos e & -r \sin b \cos e & -r \cos b \sin e \\ \sin b \cos e & r \cos b \cos e & -r \sin b \sin e \\ \sin e & 0 & r \cos e \end{bmatrix}$$

The measurement Jacobi is given by $H_k = \left.\frac{\partial h_k}{\partial X_k}\right|_{X_{k-1}}$

$$H_K = \begin{bmatrix} \frac{\delta r_r}{\delta p_x} & \frac{\delta r_r}{\delta p_y} & \frac{\delta r_r}{\delta p_z} & \frac{\delta r_r}{\delta v_x} & \frac{\delta r_r}{\delta v_y} & \frac{\delta r_r}{\delta v_z} \\ \frac{\delta b_r}{\delta p_x} & \frac{\delta b_r}{\delta p_y} & \frac{\delta b_r}{\delta p_z} & \frac{\delta b_r}{\delta v_x} & \frac{\delta b_r}{\delta v_y} & \frac{\delta b_r}{\delta v_z} \\ \frac{\delta e_r}{\delta p_x} & \frac{\delta e_r}{\delta p_y} & \frac{\delta e_r}{\delta p_z} & \frac{\delta e_r}{\delta v_x} & \frac{\delta e_r}{\delta v_y} & \frac{\delta b_r}{\delta v_z} \end{bmatrix} \tag{26}$$

Simplifying each states in (26) results to (27) for ease of notation let $p_x = x, p_y = y,$ and $p_z = z$

$$H_k = \begin{bmatrix} \frac{x_k}{\sqrt{x^2_k + y^2_k + z^2_k}} & \frac{x_k}{\sqrt{x^2_k + y^2_k + z^2_k}} & \frac{x_k}{\sqrt{x^2_k + y^2_k + z^2_k}} & 0 & 0 & 0 \\ \frac{y_k}{x^2_k + x^2_k} & -\frac{x_k}{x^2_k + x^2_k} & 0 & 0 & 0 & 0 \\ \frac{-z_k x_k}{\sqrt{x^2_k + y^2_k}(x^2_k + y^2_k + z^2_k)} & \frac{-z_k y_k}{\sqrt{x^2_k + y^2_k}(x^2_k + y^2_k + z^2_k)} & \frac{\sqrt{x^2_k + y^2_k}}{\sqrt{x^2_k + y^2_k + z^2_k}} & 0 & 0 & 0 \end{bmatrix} \quad (27)$$

8. Motion Estimation Algorithm

The flying object's trajectory is, therefore, completely specified by the initial state variables $p_x(0), p_y(0), p_z(0), \; v_x(0).v_y(0).v_z(0)$. The following is the algorithm of the research expresse in step I-IV.

I. Initialization:

Load radar data

Coordinate transformation:

[Spherical]' $\xleftarrow[ECEF]{RAE}$ [Rectangular]'

Initialize filter state $x_k\,(0)$ given by [$p'(0), v'(0)$]'.

II. The free-flight measurement is used to estimate $x_k^{ff}(k)$ by solving the vector Θ given by [8]

$$\Theta = \begin{bmatrix} \hat{X}(k) \\ position' \\ velocity' \end{bmatrix} = x_k^{ff}(k) = \arg\min_{\hat{x}_t^0} \sum_{t-N}^{N-1} \| Z_k - h_k(\varphi(\hat{x}_k^{ff})) \|^2_R$$

(28)

By using the initial conditions $\hat{x}(0)$ from (29).

III. Then, the estimate $x_k^{ff}(k)$ is obtained by propagating the estimated states up to time 375s, i.e.

$x_k(k) = \psi(\hat{X}_k)$

IV. The corrected estimates, update, and actual values are then shown and plotted

9. Simulation Result Analysis

This paper has simulated BM trajectory with a radar data rate of 2 Hz. The BM trajectory is shown in Figure 5, the trajectory estimated by EKF can better fit the actual trajectory, and the uncertainty is minimized relative to the trajectory directly recorded by the observed value.

The ground-based radar is located at the ENU origin. The model is used to put Gaussian noise on the observations with standard deviations of 0.001rad for both azimuth and elevation, and 10 meters for the range.

The Initial conditions are;

$$\begin{bmatrix} p_x(0) & p_y(0) & p_z(0) \end{bmatrix} = \begin{bmatrix} -766 & -43 & 69 \end{bmatrix} (km)$$

$$\begin{bmatrix} V_x(0) & V_y(0) & V_z(0) \end{bmatrix} = \begin{bmatrix} 1.8 & -0.06 & 1.8 \end{bmatrix} (km/s)$$

$$Q = \begin{Bmatrix} 10^{4} & 10^{4} & 10^{4} & 10^{4} & 10^{4} & 10^{4} \\ 10^{-5} & 10^{-5} & 10^{-5} & 10^{-5} & 10^{-5} & 10^{-5} \end{Bmatrix} \begin{matrix} \\ for \\ \end{matrix} \begin{matrix} k < 10s \\ k > 10s \end{matrix} \quad (29)$$

Total track duration is 375s. The range measurement error deviation and the process noise level are tuned according to the EKF. Finally, the sampling interval (T) is 0.5s (which means that the number of measurements is twice in seconds).

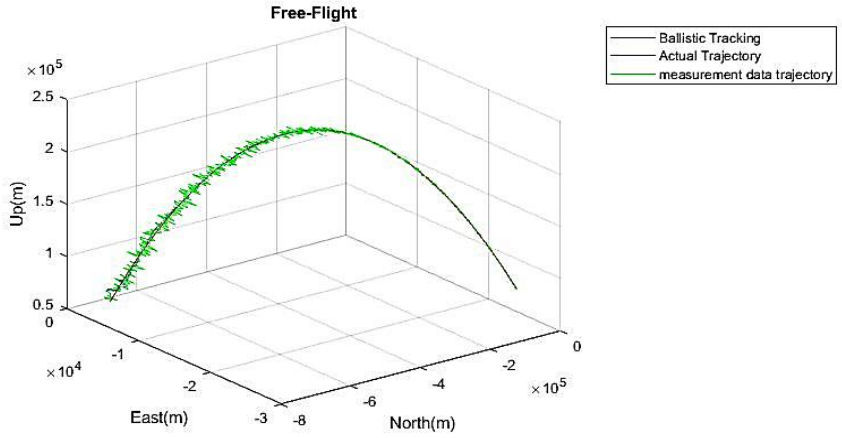


Figure 3: Simulation Result of Ballistic missile trajectory.

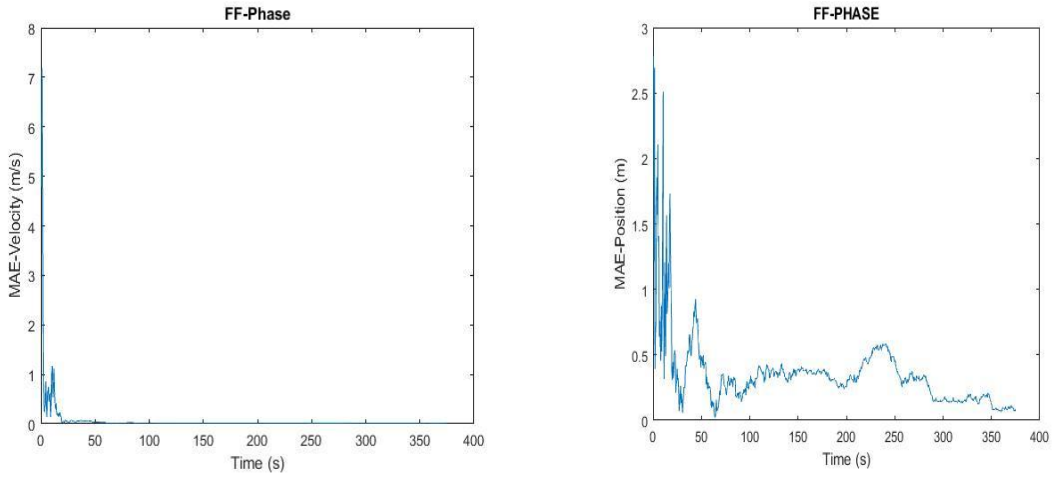
10. Evaluation Criteria

MAE is chosen here as the evaluation criteria, is the measures of the average magnitude of error in sets of predictions, without considering their direction. Is the average over the test sample of the absolute diffrences between prediction and actual observation where all individual diffrences have equal weight. given by the following

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|X_i - \hat{X}_i\right| \quad (30)$$

Let the target state vector be *i*, The true value of the dimension X_i , Estimated value $\hat{X}_i$

Then the filtering performance of the EKF algorithm is shown in Fig. 4a and 4b. It can be seen that the MAE is not monotonically decreasing because the new parameters are also being estimated while receiving new data samples, and the uncertainty of the new parameters added due to the influence of the driving noise input may be large enough to offset the knowledge gained from the new sample data, resulting in an increase in MAE.



a **b**

Figure 4: shows the relationship between velocity (MAE) and time, while 4b Relationship between position (MAE) and Time.

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The author would like to acknowledge my late mum, B. Adaora, my teacher and also thanks to Prof. Elijah Omizegba for this brilliant contribution.

CONCLUSION

In this paper, we proposed the algorithm for estimating the ballistic missile trajectory using radar measurments. In the development of ballistic missile defense in the free-flight segment, the important tracking problem by radar of BM system has been studied based on EKF, where both the uncertainty in modeling and the noise in the system were taken into account. The simulation result proves its validity.

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APPENDIX

$$F_{4,1}^{ff} = -\frac{\mu}{p^3} + \frac{3x^2\mu}{p^5}$$

$$F_{4,2}^{ff} = \frac{3xy\mu}{p^5}$$

$$F_{4,3}^{ff} = \frac{3xz\mu}{p^5}$$

$$F_{4,4}^{ff} = -\frac{p(h)V}{2\beta} - \frac{p(h)v_x}{2V\beta}$$

$$F_{4,5}^{ff} = -\frac{v_x v_y p(h)}{2V\beta}$$

$$F_{4,6}^{ff} = -\frac{v_x v_y p(h)}{2V\beta}$$

$$F_{5,1}^{ff} = \frac{3xy\mu}{p^5}$$

$$F_{5,2}^{ff} = -\frac{\mu}{p^3} + \frac{3y^2\mu}{p^5}$$

$$F_{5,3}^{ff} = \frac{3yz\mu}{p^5}$$

$$F_{5,4}^{ff} = -\frac{v_x v_y p(h)}{2V\beta}$$

$$F_{5,5}^{ff} = -\frac{p(h)V}{2\beta} - \frac{p(h)v_y}{2V\beta}$$

$$F_{5,6}^{ff} = -\frac{v_z v_y p(h)}{2V\beta}$$

$$F_{6,1}^{ff} = \frac{3xz\mu}{p^5}$$

$$F_{6,2}^{ff} = \frac{3yz\mu}{p^5}$$

$$F_{6,3}^{ff} = -\frac{\mu}{p^3} + \frac{3z^2\mu}{p^5}^2$$

$$F_{6,4}^{ff} = -\frac{v_x v_z p(h)}{2V\beta}$$

$$F_{6,5}^{ff} = -\frac{v_y v_z p(h)}{2V\beta}$$

$$F_{6,6}^{ff} = -\frac{p(h)V}{2\beta} - \frac{p(h)v_z^2}{2V\beta}$$

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